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May 28, 2007

Notes on
Gould, "Rising Wage Inequality, Comparative Advantage, and the Growing Importance of General Skills in the United States"

Stylized Facts about Growth in Wage Inequality

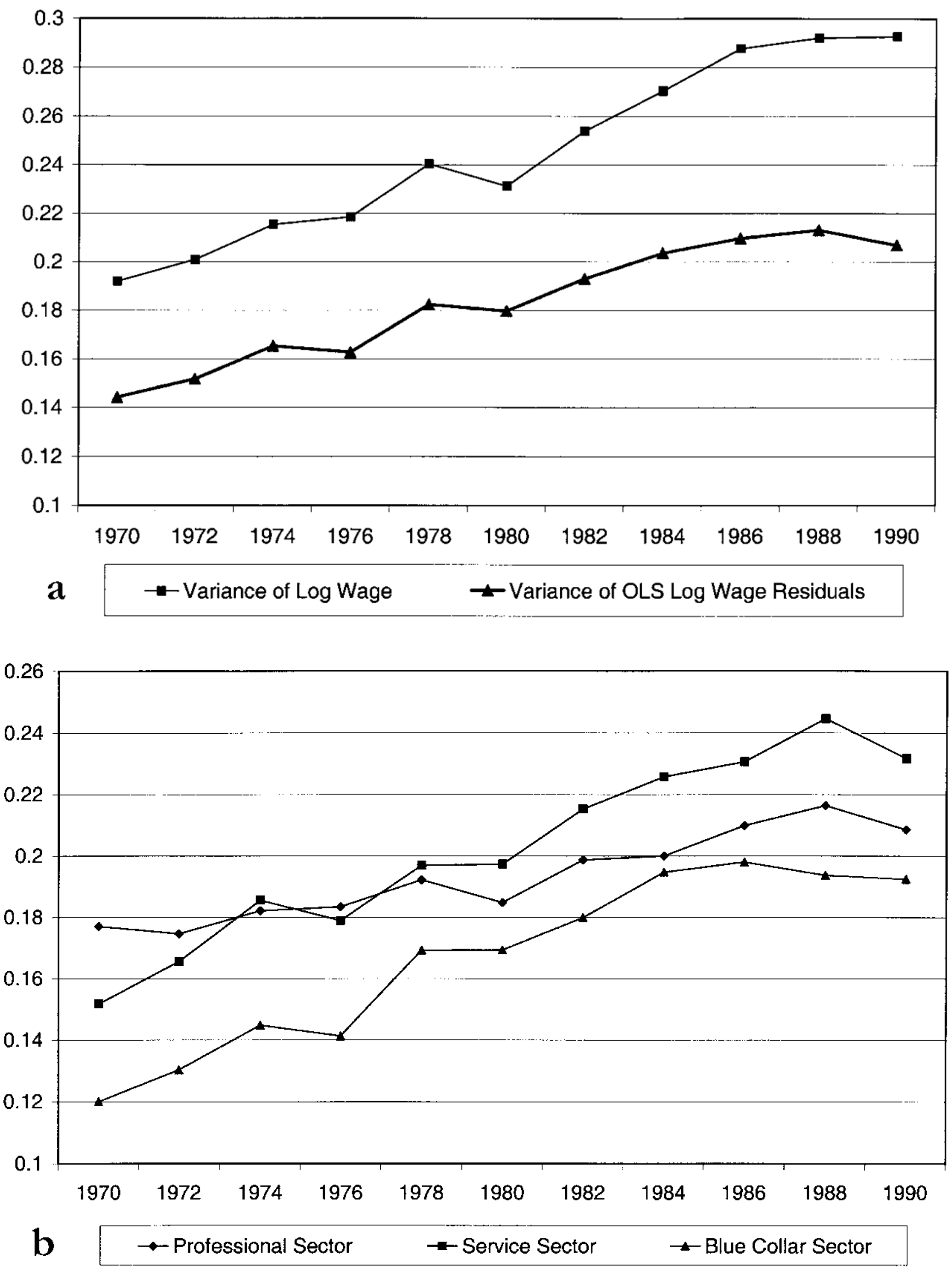


Fig. 1.— *a*, Variance of log wages. Uniform March Current Population Survey data. Residuals were computed for white men after controlling for years of schooling, experience, region of residence, marital status, and living in a standard metropolitan statistical area. See Sec. II for data construction and variable definition. *b*, Variance of ordinary least squares wage residuals within occupational sectors. Uniform March Current Population Survey data. Residuals were computed for white men after controlling for years of schooling, experience, region of residence, marital status, and living in a standard metropolitan statistical area. See Sec. II for data construction and variable definition.

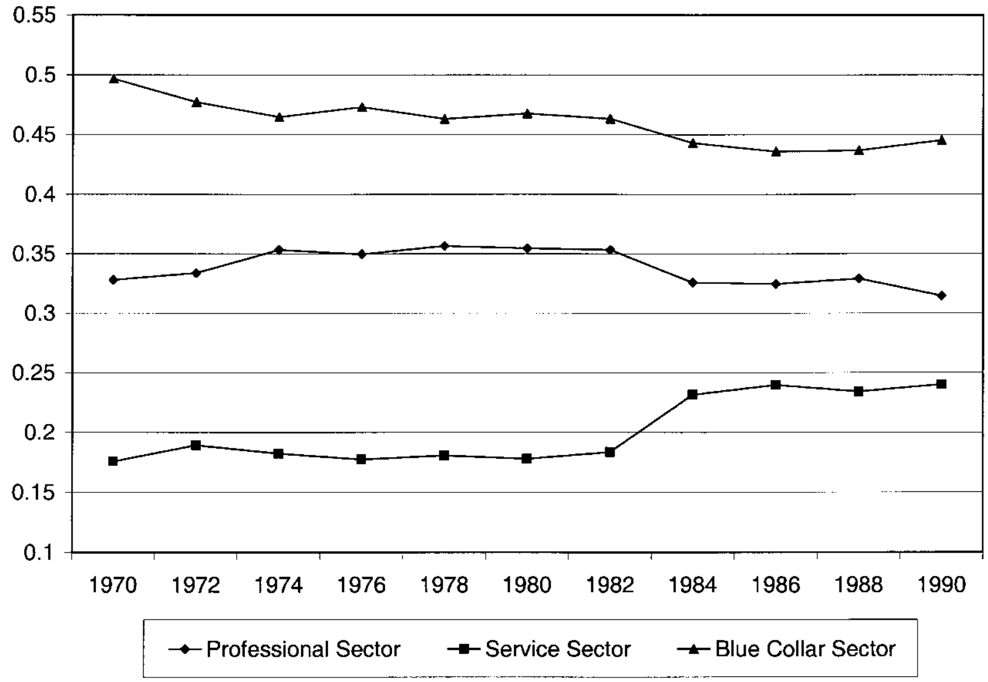


Fig. 2.— Proportion of workers in each occupation over time. Uniform March Current Population Survey data. See Sec. II for data construction and variable definitions.

Model

that a worker chooses to work in sector i over sector j if

$$\pi_i T_i(x) \geq \pi_j T_j(x),\ i \neq j;\ i, j = 1,\ 2. \tag{1}$$

The log wage in sector i for a worker with skill endowment x is given by

$$\begin{aligned} \ln w_i(x) &= \ln \pi_i + \ln T_i(x) \\ \ln w_i(x) &= \ln \pi_i + t_i(x), \end{aligned} \tag{2}$$

where $t_i(x)$ is the natural log of the function $T_i(x)$. Following Roy, assume that the distributions of skill endowments and the task functions are such that the distributions of abilities are log-normally distributed. Specifically, the population distribution of log ability in sector i is characterized by

$$t_i \sim N(\mu_i,\ \sigma_{ii})\ i = 1,\ 2,$$

where the covariance between the log ability to be a lawyer and an athlete is given by σ_{12}. Log wages in each sector can then be written as

$$\begin{aligned} \ln w_1 &= \ln \pi_1 + \mu_1 + u_1 \\ \ln w_2 &= \ln \pi_2 + \mu_2, + u_2, \end{aligned} \tag{3}$$

where u_1 and u_2 are normally distributed random variables with a mean

$$E(\ln w_i | \ln w_i \geq \ln w_j) = \ln \pi_i + \mu_i + E(u_i | \ln w_i \geq \ln w_j) \quad i \neq j,$$

where $E(u_i | \ln w_i \geq \ln w_j) \neq E(u_i) = 0$ because of the nonrandom selection. This expression can be rewritten as

$$\begin{aligned} & E(\ln w_i | \ln w_i \geq \ln w_j) \\ & = \ln \pi_i + \mu_i + E(u_i | \ln \pi_i - \ln \pi_j + \mu_i - \mu_j \geq u_j - u_i) \\ & = \ln \pi_i + \mu_i + [(\sigma_{ii} - \sigma_{ij})/\sigma^*]\lambda(c_i), \end{aligned} \tag{4}$$

where $\sigma^* = \sqrt{\mathrm{Var}(u_i - u_j)}$, $c_i^* = \ln \pi_i - \ln \pi_j + \mu_i - \mu_j$, $c_i = c_i^*/\sigma^*$, and $\lambda(c_i) = \phi(c_i)/\Phi(c_i)$, where $\phi(x)$ and $\Phi(x)$ are the standard normal probability density function (pdf) and cumulative distribution function (cdf), respectively, evaluated at x. The variance of log wages, given that sector i is chosen, can be written as

$$\mathrm{Var}(\ln w_i | \ln w_i \geq \ln w_j) = \sigma_{ii}\{1 - \rho_i^2[c_i\lambda(c_i) + \lambda^2(c_i)]\}, \tag{5}$$

where ρ_i is equal to the correlation between u_i and $(u_i - u_j)$. The conditional variance must be less than the unconditional variance since the expression in brackets is less than one. If we divide the conditional variance by the unconditional variance, we can determine the percentage that the conditioning information (comparative advantage) has reduced the unconditional variance with the following formulation:

$$S_i = \{1 - \rho_i^2[c_i\lambda(c_i) + \lambda^2(c_i)]\}, \tag{6}$$

where S_i is less than one. The expression $(1 - S_i)$ measures the percentage contribution of comparative advantage in reducing the unconditional population variance in sector i.

We can also construct the counterfactual; that is, the expected log wage in sector j for the workers observed in sector i is

$$\begin{aligned} & E(\ln w_j | \ln w_i \geq \ln w_j) \\ & = \ln \pi_j + \mu_j + E(u_j | \ln \pi_i - \ln \pi_j + \mu_i - \mu_j \geq u_j - u_i) \\ & = \ln \pi_j + \mu_j - [(\sigma_{jj} - \sigma_{ij})/\sigma^*]\lambda(c_i). \end{aligned} \tag{7}$$

Since $w_i = \pi_i T_i$, we can define the relative ability in sector i of the people who choose sector i as the differential between the mean log ability in sector i of workers who choose sector i and the population mean log ability in sector i:

$$R_i^i = E(t_i | \ln w_i \geq \ln w_j) - \mu_i = [(\sigma_{ii} - \sigma_{ij})/\sigma^*]\lambda(c_i). \tag{8}$$

The relative ability of workers who choose sector i in sector j is defined similarly:

$$R^i_j = E(t_j | \ln w_i \geq \ln w_j) - \mu_j = -[(\sigma_{jj} - \sigma_{ij})/\sigma^*]\lambda(c_i). \tag{9}$$

This measure indicates whether any particular sample possesses above- or below-average ability in any particular sector and measures the percentage magnitude. If the workers who select sector i possess above-average ability in that sector (i.e., $R^i_i > 0$), this is referred to as "positive selection" in sector i. Negative selection occurs if the workers who choose sector i are below average (i.e., $R^i_i < 0$). If $R^i_i = 0$, then there is no selection in the sense that workers who choose sector i appear as if they were chosen at random. By substituting in for the specific sectors, we have the relative abilities for the samples in their observed sectoral choices:

$$R^1_1 = [(\sigma_{11} - \sigma_{12})/\sigma^*]\lambda(c_1)$$

$$R^2_2 = [(\sigma_{22} - \sigma_{12})/\sigma^*]\lambda(c_2).$$

We can write the counterfactual relative abilities as

$$R^1_2 = -[(\sigma_{22} - \sigma_{12})/\sigma^*]\lambda(c_1)$$

$$R^2_1 = -[(\sigma_{11} - \sigma_{12})/\sigma^*]\lambda(c_2).$$

It is important to note that $\lambda(c_i)$ for $i, j = 1, 2$ is always positive and is a decreasing function of c_i, where c_i is increasing in the relative price and relative mean of ability in sector i.

Implications of Model for Earnings Inequality and Ability Sorting:

Focus of paper on possibility of technological change over time and its consequences for earnings inequality.

Technological change that affects variances of ability in occupations:

"Dumbing Down" or "Anyone-can-do-it" effect: Technology replaces human skills and collapses distribution of ability around mean.

If this is what technological change does, it would decrease the magnitude of positive selection in an occupation.

PROPORTION 1. A relative decrease (increase) in a sector's variance tends to reduce (increase) the positive selection (bias) in that sector.

Technological change that affects covariance of abilities:

If technological change makes certain type of skills, e.g., analytic and computer skills, more important in all jobs, then covariance of ability in many or all sectors increases (becomes more positive).

PROPORTION 2. An increase (decrease) in the covariance in abilities across sectors tends to decrease (increase) the positive selection (bias) in sector i.

Implications of Self-Selection for Observed Variances of Abilities:

PROPORTION 3. As workers choose their sectors according to their comparative advantage, the resulting overall wage dispersion will be less than what would result from a randomly assigned economy.

Technological change through changes in relative prices or relative population means of ability:

PROPORTION 4. A relative increase (decrease) in sector i's population mean of ability or task price will increase (decrease) sector i's inequality and decrease (increase) sector j's inequality as workers leave sector j from the low end (of the ability distribution) and enter sector i on the low end.

Estimation

data described in Section II. Let i ($i = 1, \ldots, N$) index each individual and j ($j = 1, 2, 3$) index the occupational choice set. For any given year, each individual chooses their sector by utility maximization, where the utility of individual i in sector j is represented as follows:

$$U_{ij} = \beta_j Z_i + \gamma_j W_{ij} + v_{ij},$$

where

Z_i is an $L_z \times 1$ vector of observable, exogenous variables for person i in all three sectors,

β_j is a $1 \times L_z$ utility parameter vector on the exogenous variables in sector j,

W_{ij} is the log wage of person i in sector j,

γ_j is the sector j utility parameter on the log wage received in sector j,

v_{ij} is an independent (across individuals, sectors, and years) and identically normally distributed stochastic component of utility for person i in sector j, with a mean of zero and a variance equal to $\sigma^2_{v_j}$.

The log wage for individual i in sector j is modeled by the following:

$$W_{ij} = \delta_j X_i + \sigma_j f_i + u_{ij},$$

where

X_i is an $L_x \times 1$ vector of observable, exogenous variables for person i that enters all three sectors,

δ_j is a $1 \times L_x$ vector of parameters on the exogenous variables,

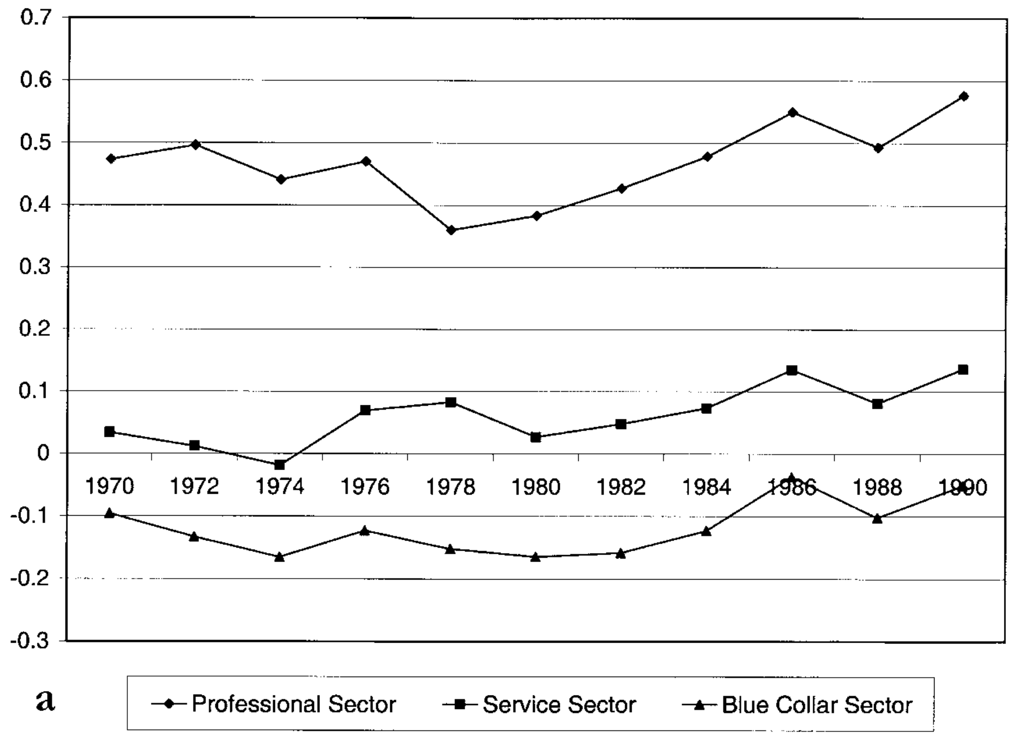
f_i is a scalar random factor distributed (either normally or uniformly) with a mean of 0.5 and a variance equal to σ^2_{f},

σ_j is a scalar sector-specific factor loading,

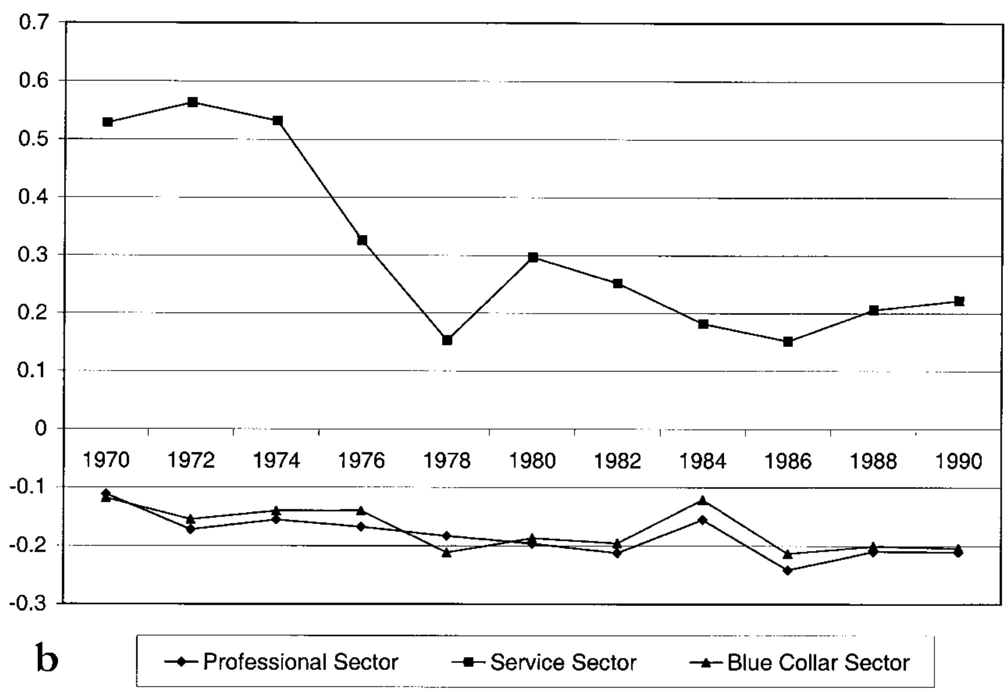
u_{ij} is an independent (across individuals, sectors, years, and from f_i and v_{ij}) and identically normally distributed stochastic component of utility in sector j for person i, with a mean of zero and a variance equal to $\sigma^2_{U_j}$.

where f_i is "general" unobservable skill and u_{ij} is "sector-specific" unobs. skills, and σ_j for sector j measures importance of general skill in sector j's production and, thus, wages.

Relative Ability of Workers

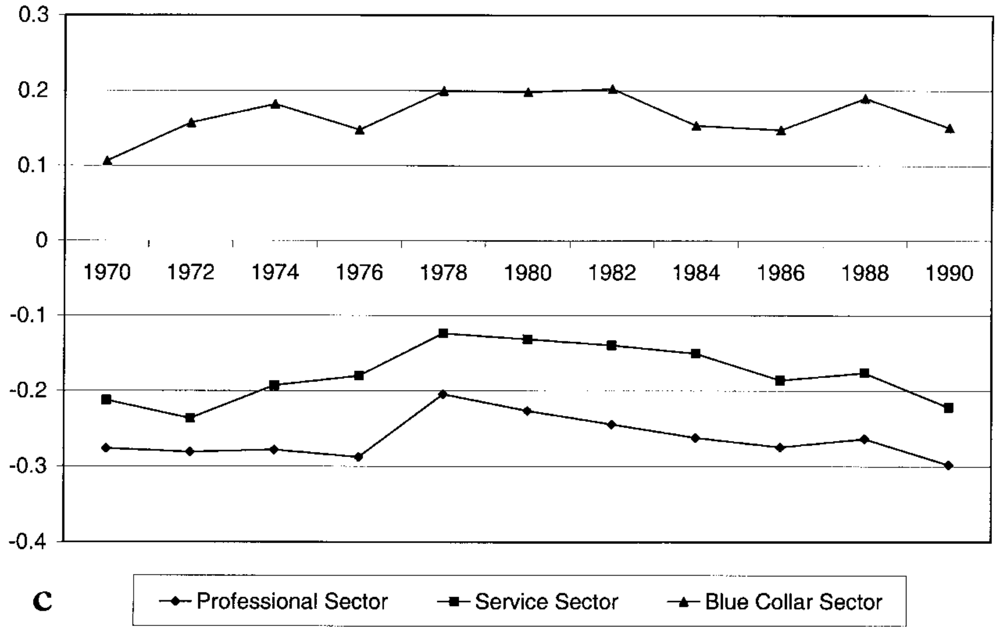


a



b

Fig. 3.—*a*, Relative abilities of professional workers in each sector. *b*, Relative abilities of service workers in each sector.



c

Fig. 3.—*c*, Relative abilities of blue-collar workers in each sector

Population Sectoral Variances in Abilities

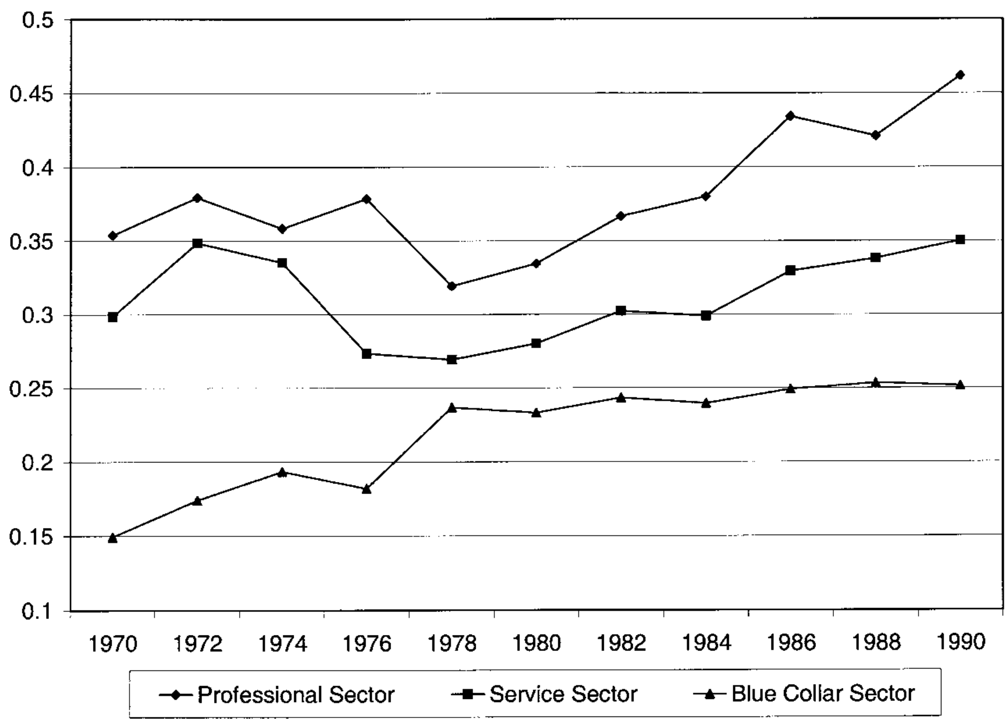


Fig. 4.—Population variance of ability in each sector

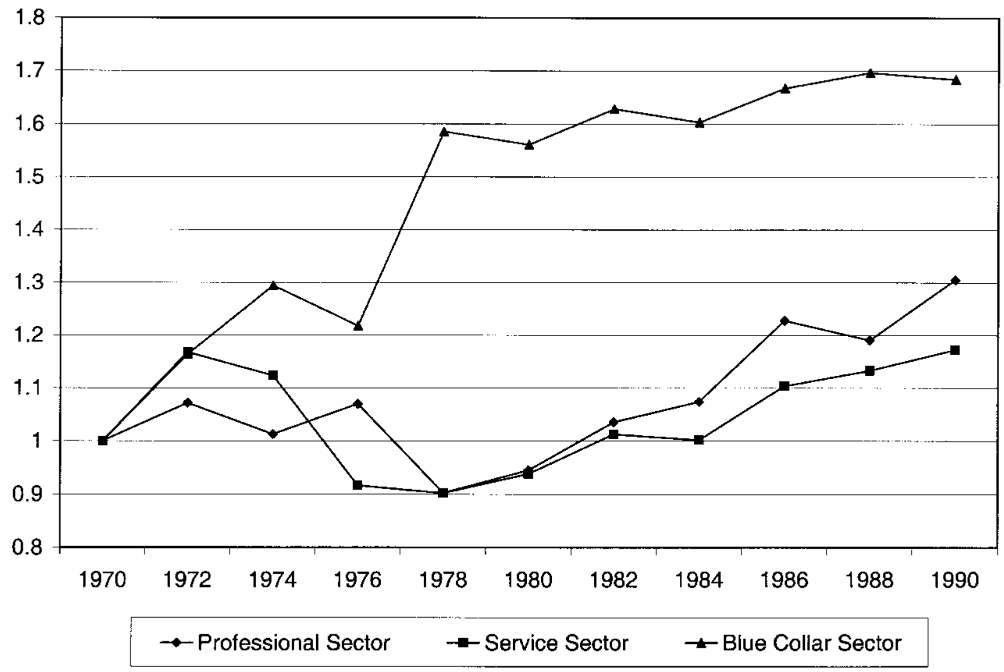


Fig. 5.—Changes in the hierarchy of sectors over time. The estimated population variances of ability in each sector from fig. 4 are standardized to the base year, 1970.

Population Covariances in Sectoral Abilities

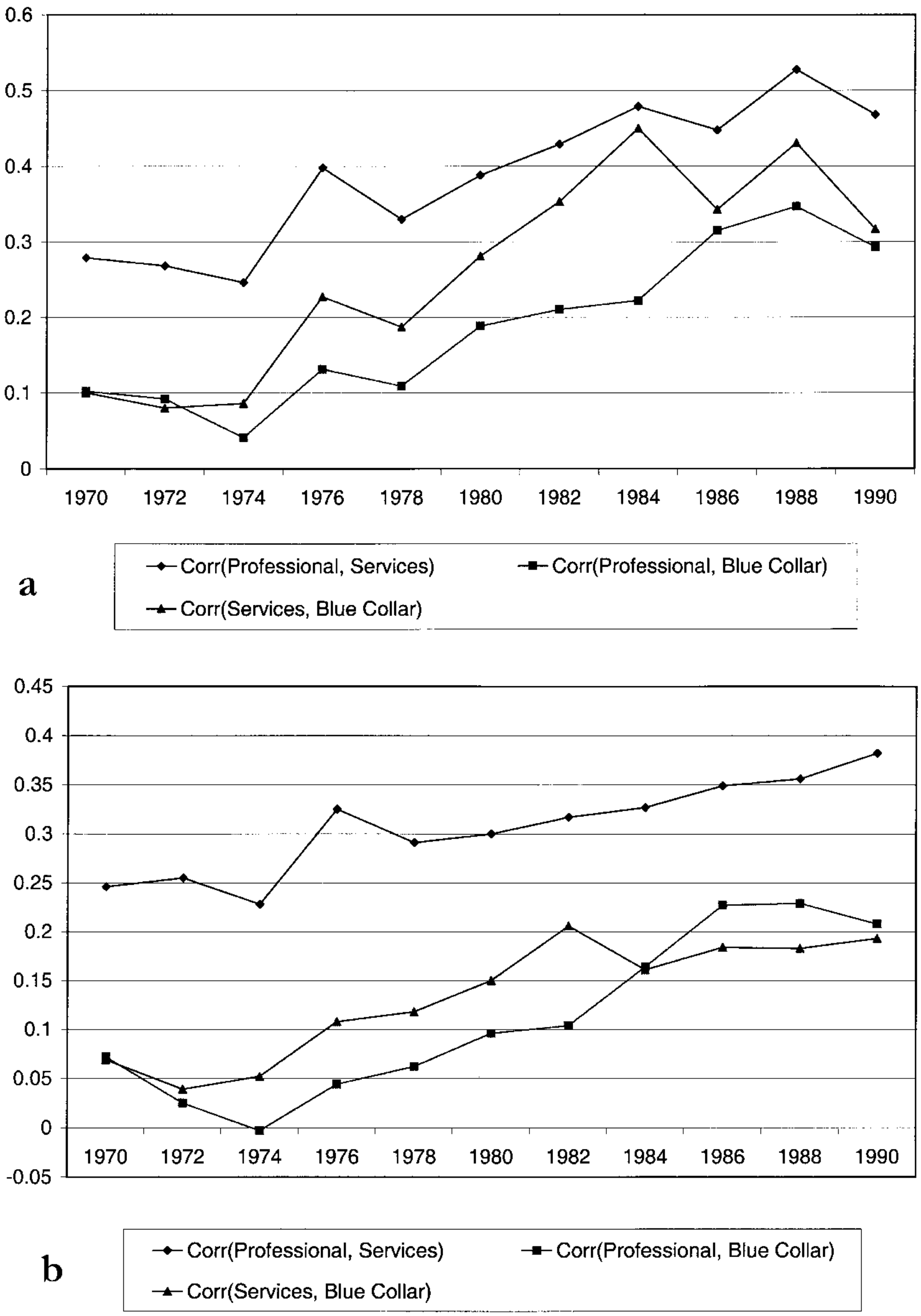


Fig. 6.—*a*, Correlations of abilities across occupational sectors (normal specification). Estimates are from the model specification where the general factor is assumed to be distributed normally, as described in app. A. *b*, Correlations of abilities across occupational sectors (uniform specification). Estimates are from the model specification where the general factor is assumed to be distributed with a uniform distribution, as described in app. B.

Sectoral Task Prices and Sectoral Mean Abilities

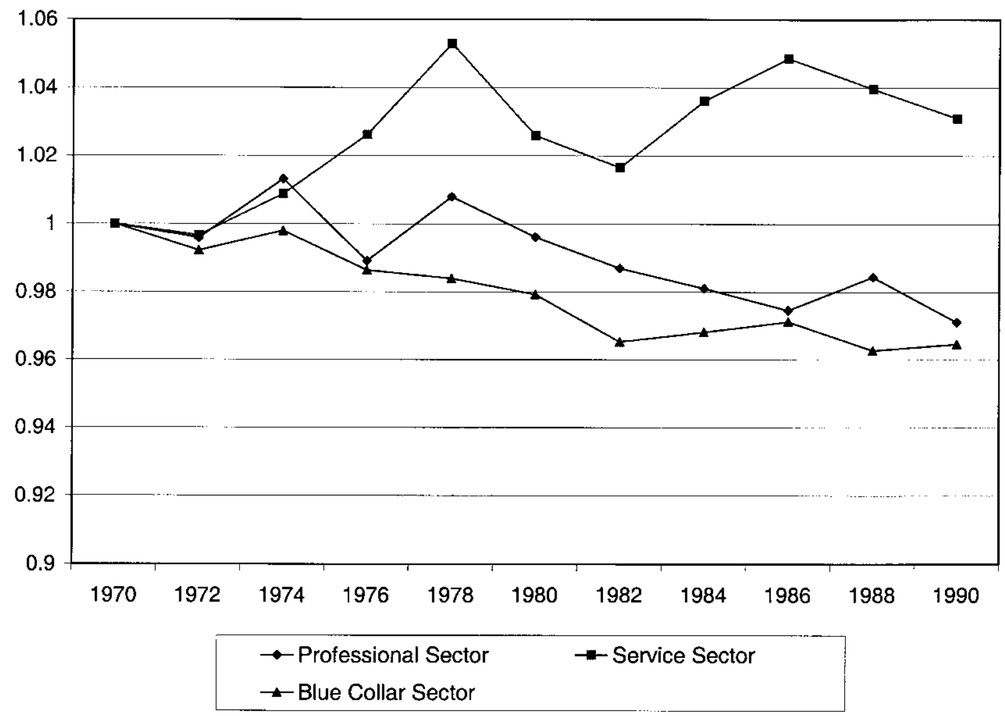
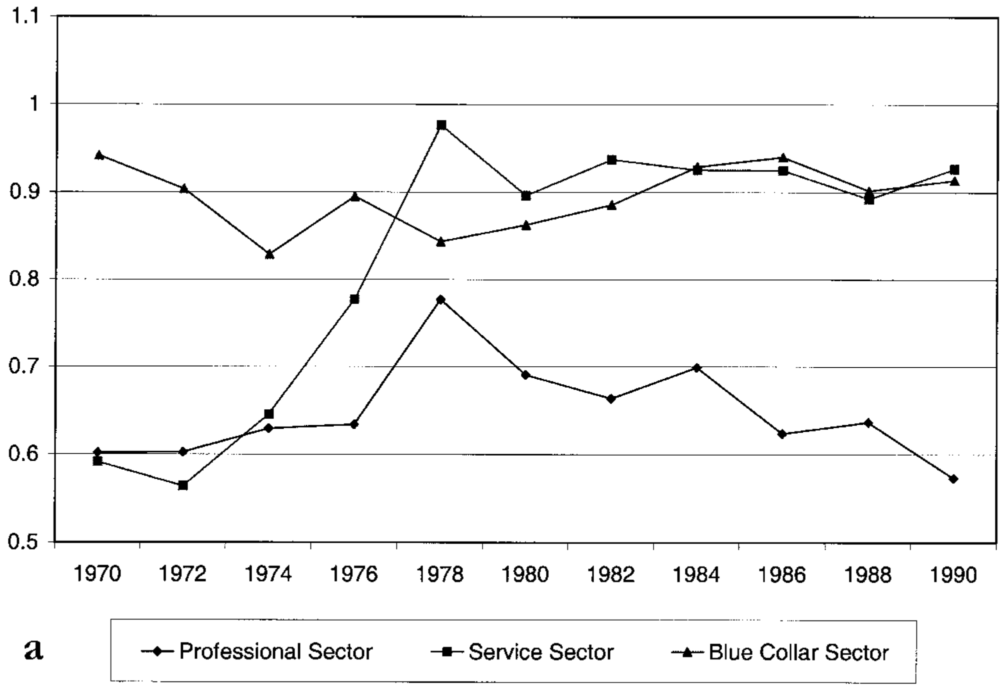
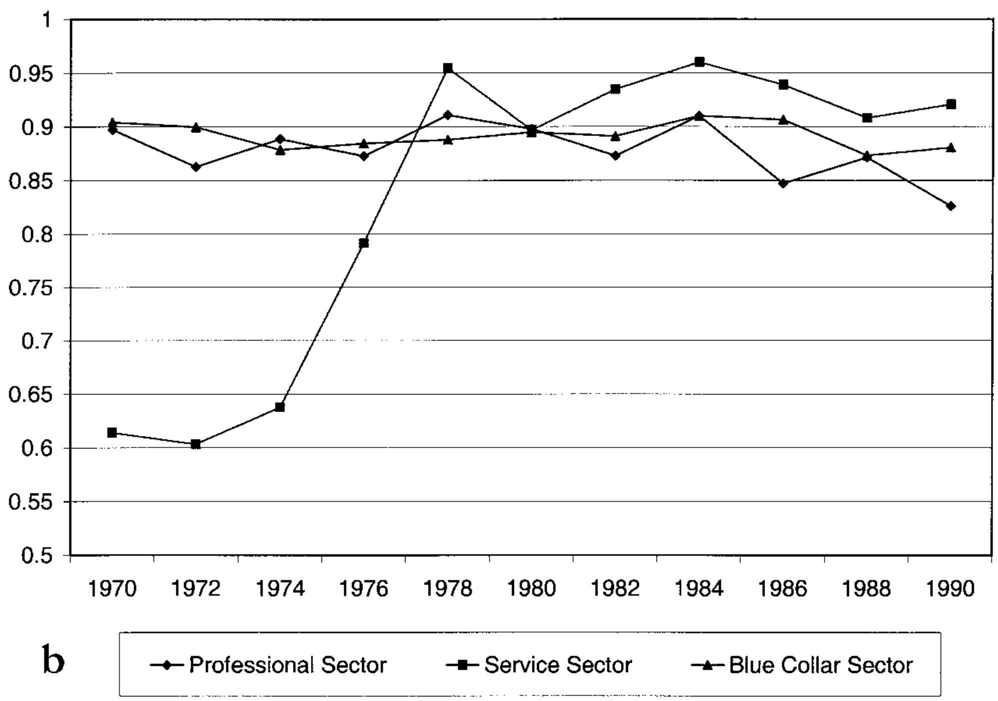


Fig. 7.—Sum of the task price plus mean ability for each sector. For each sector, the estimated sum is standardized to unity in 1970.

Role of Comparative Advantage in Reducing Inequality



a



b

Fig. 8.—*a*, The contribution of self-selection to inequality within each sector. For each sector, the graph represents the ratio of the variance of wages for those who self-select into that sector to the variance of wages in the population for that sector (the self-selection variance over the random assignment variance). *b*, The contribution of self-selection to the variance of unobserved ability within each sector. For each sector, the graph represents the ratio of the variance of unobserved ability (residual wage variance) for those who self-select into that sector to the variance of unobserved ability in the population for that sector (the self-selection variance over the random assignment variance).

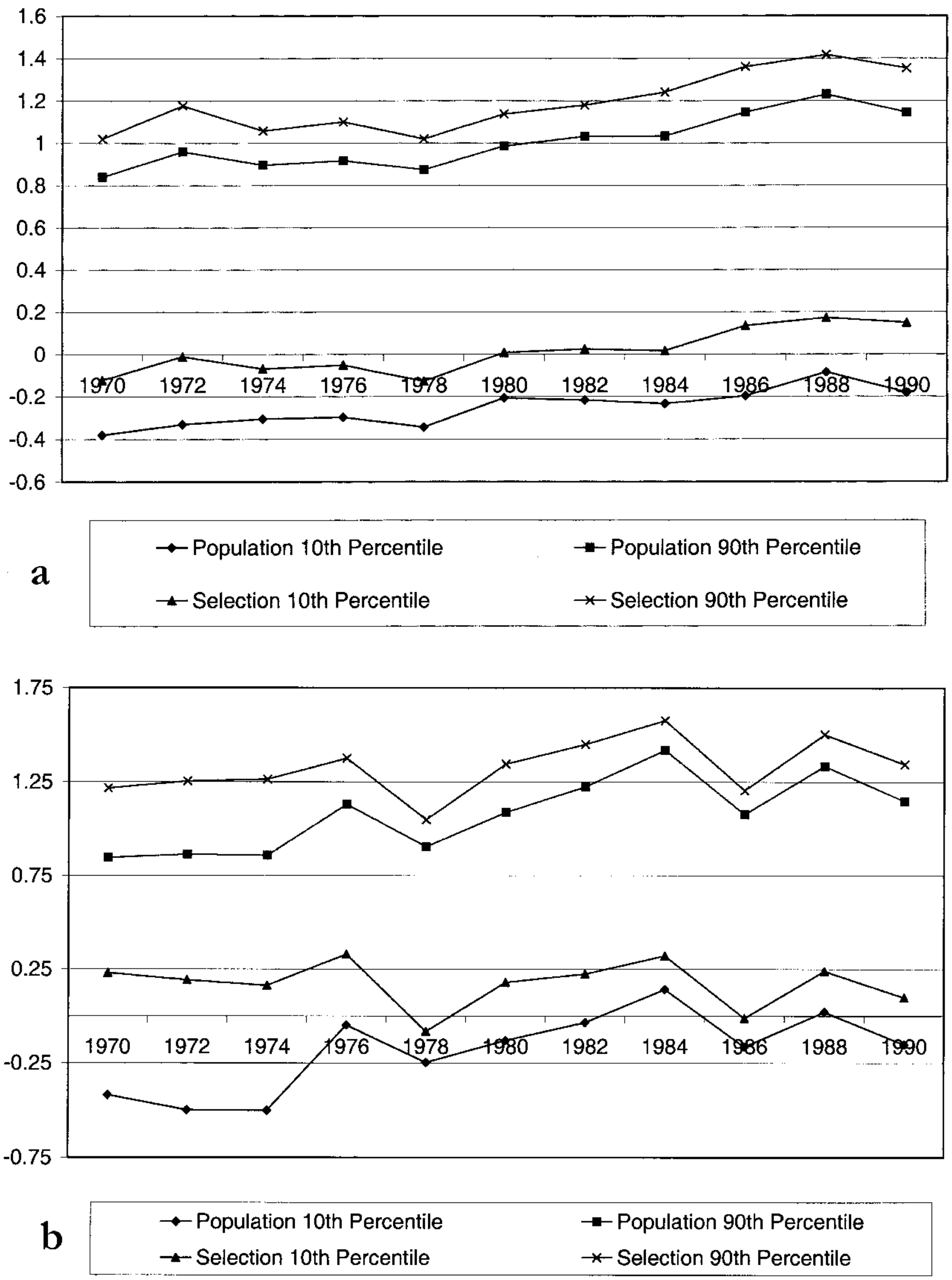
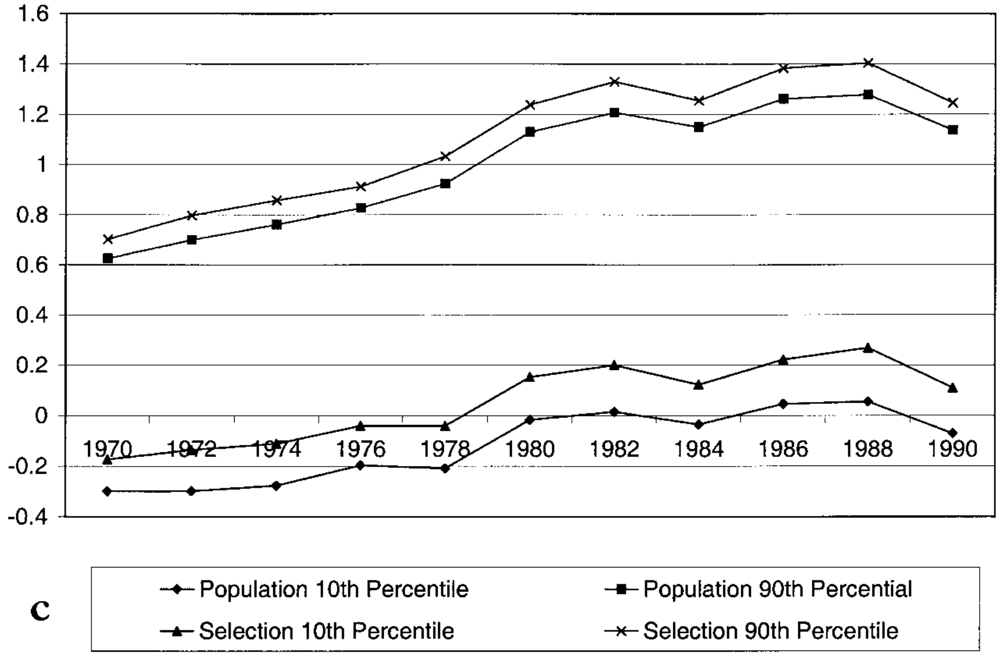


Fig. 9.—*a*, Percentiles of the self-selected and population distributions of unobserved ability in the professional sector. *b*, Percentiles of the self-selected and population distributions of unobserved ability in the service sector.



c

Fig. 9.—*c*, Percentiles of the self-selected and population distributions of unobserved ability in the blue-collar sector.

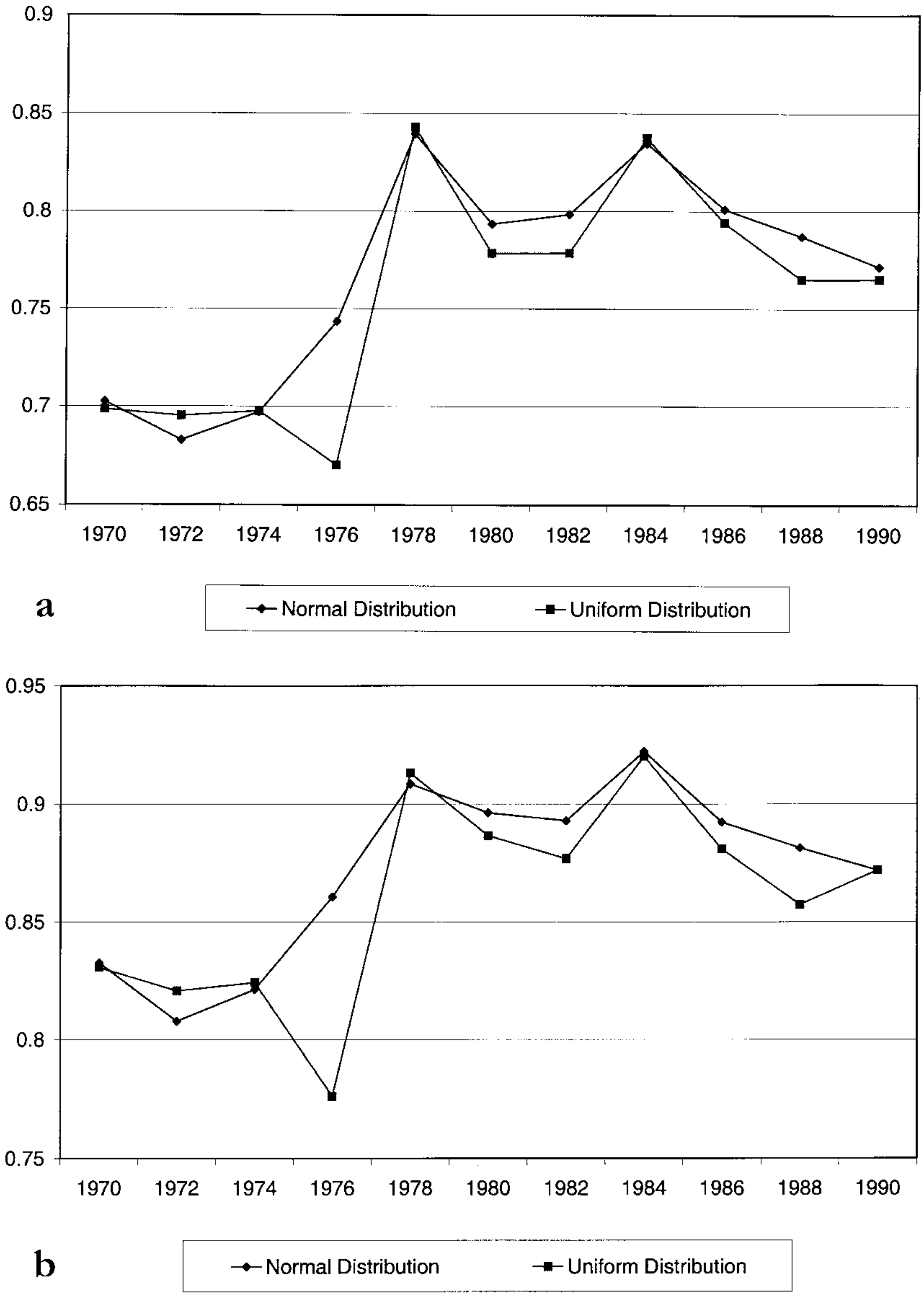


Fig. 10.—*a*, The importance of self-selection in reducing total within-sector inequality. For the entire sample, the graph represents the ratio of the total within-sector variance of wages after workers sort themselves (self-select) into sectors to the total within-sector variance of wages in an economy where workers are sorted into occupations at random. Each line refers to model estimates based on the distributional assumption of the general unobserved factor, as described in app. A. *b*, The importance of self-selection in reducing total within-sector unobservable inequality. For the entire sample, the graph represents the ratio of the total within-sector variance of unobservable skills after workers sort themselves (self select) into sectors to the total within-sector variance of unobservable skills in an economy where workers are sorted into occupations at random. Each line refers to model estimates based on the distributional assumption of the general unobserved factor, as described in app. A.