Some Suggested Improvements to a Simple Portfolio Balance Model of Exchange Rate Determination with Special Reference to the U.S. Dollar/Canadian Dollar Rate

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I.

Perhaps the simplest portfolio balance model of exchange rates is the one suggested by Branson *et al.* [1977][1]. Formally it involves only a single country with three assets: non-interest bearing money, an interest bearing domestic asset and an interest bearing, internationally tradeable asset. This model is only plausible under very restrictive assumptions. The purpose of this note is to set out the structure and assumptions of Branson *et al.*'s formal model; to criticize both the formal model and the estimation procedures and to propose a more general model which is largely free of those criticisms; finally, with reference to the U.S. dollar/Canadian dollar exchange rate, to present some preliminary empirical results.

The structure of Branson *et al.*'s model is as follows:

$$M = m(i, i^*)\,W \tag{1}$$

$$B = b(i, i^*)\,W \tag{2}$$

$$eF = f(i, i^*)\,W \tag{3}$$

$$M + B + eF = W \tag{4}$$

where M is money, B is domestic bonds, F is a foreign asset denominated in foreign currency, e is the exchange rate defined as the domestic currency price of foreign exchange, i is the rate of interest on B, i* is the

Remark: The views expressed in this paper are those of the authors and not necessarily those of the Federal Reserve Bank of San Francisco or the Federal Reserve System.

[1] For other expositions see Branson [1977]; Branson and Halttunen [1979]; Bisignano and Hoover [1980]. — The portfolio balance models used in Bisignano and Hoover are in the spirit of those of Branson *et al.* The criticisms made in this note apply equally to them.

rate of interest on F, and W is total wealth defined by the identity (4). M and B are assumed to be non-traded assets. The desired fraction of wealth held as money is m, held as domestic bonds, b, and held as foreign assets denominated in domestic currency, f.

This is a model of private portfolio selection. All the assets are held in private portfolios. Branson and his colleagues add the refinements of government reaction functions and expectations schemes. Such additions are necessary for a complete model of exchange rates, but they do not concern us here. (Because this may involve some bias in the econometric analyses, a particular estimation scheme will be proposed which should reduce any simultaneous equations bias.)

In order for this model to be empirically useful, two assumptions are made: first, that the economy that it describes is "small" — i.e., that it cannot affect "world" interest rates; second, that there is just one internationally exchangeable asset (exchangeable for goods and services but not for other financial assets)[1]. A third, fundamental assumption is that there is continuous equilibrium in financial markets. This is a model of extremely short-run behavior. As such, stock equilibrium is assumed on the basis that the time interval is short enough to disregard flow market conditions, such as the state of the current account. If the time interval is inappropriate for the stock equilibrium assumption, the formal model would have to build in explicit disequilibrium conditions.

If the asset stocks — M, B, and F — are taken as given at any time, the model has three variables, the two interest rates and the exchange rate, but because of the identity (equation (4)) only two independent equations. The small-country assumption allows one to take i^* as parametrically given, that is as exogenously determined, thus eliminating one variable and rendering the system mathematically determined.

With the benefit of the small-country assumption, the comparative statics yield testable hypotheses. An expansion in the domestic money stock is expected to depreciate the value of the domestic currency and an increase in the stock of foreign bonds to cause an appreciation. An expansion in the supply of domestic bonds may either appreciate or depreciate the currency, depending on the relative substitutability between domestic bonds and money versus domestic and foreign bonds.

[1] Frankel [1981, p. 24] characterizes the small-country assumption as holding when foreigners do not hold the domestic bonds of the small country. This is an inadequate criterion, however, for if the small country in acquiring more or giving up foreign assets affects the foreign rate of interest, that rate is not exogenous with respect to the small country's portfolio as the Branson model requires, irrespective of whether it issues its own assets overseas.

The model has been estimated with the following reduced form for the exchange rate[1]:

$$e_t = \Phi(M_t, B_t, F_t, M_t^*, B_t^*, F_t^*) + u_t \qquad (5)$$

where the starred variables are the similar asset stocks to the unstarred variables, but for the foreign country, and u is a random error term.

II.

The problems of the model in equations (1)—(4) are not in the formal structure. One can imagine a world in which such a model would be appropriate. They are rather in applying this model to the world at hand. Obviously all models involve simplification. The only question is whether or not in simplifying essential features are ignored or altered. Let us consider in this light several difficulties with the model.

The usual practice of estimating the reduced form in equation (5) implies an additional assumption to those noted in Section I — namely, that the world for purposes of estimation consists of just two countries or, more precisely, that third country internationally tradeable assets are perfect substitutes for F *and vice versa* in the portfolios of the home country. This is evident in the fact that e is usually taken to be a bilateral exchange rate and not some weighted index or effective exchange rate against the rest of the world. More important is the general practice of using the *net* international investment position of a country to serve as F or F*. This net position comprises the assets and the liabilities of each country against all others. Unless the assumption of perfect substitutability of third country assets holds, a multilateral model is required.

It is not immediately clear from equations (1)—(4) and the assumptions outlined above why the reduced form includes the starred variables. If the small-country assumption is taken seriously only i*, an exogenous interest rate would appear in addition to the home country's asset stocks in the reduced form. One way of justifying the form of equation (5), however, is to suppose that i* is determined by a reduced form:

$$i_t^* = \theta(M_t^*, B_t^*, F_t^*) + w_t \qquad (6)$$

assuming that there are indeed but two countries and that the other one is small. Assume w_t and v_t to be stochastic error terms of their respective

[1] See Bisignano and Hoover [1980]. Branson *et al.* [1977, p. 311], in fact, drop the domestic bond stocks, B and B*, for the econometrically spurious reason that the sign of their coefficients cannot be determined a priori. Whether or not the sign of the coefficient is known in advance, an omitted variable biases the regression coefficients.

reduced form equations, (6) and (7). Then, if the proper reduced form of e from equations (1)—(4) is:

$$e_t = \psi\,(M_t, B_t, F_t, i_t^*) + v_t \qquad (7)$$

equation (6) may be substituted into equation (7) yielding

$$e_t = \psi\,(M_t, B_t, F_t, \theta\,(M_t^*, B_t^*, F_t^*)) + w_t + v_t \equiv$$
$$\Phi\,(M_t, B_t, F_t, M_t^*, B_t^*, F_t^*) + u_t \qquad (5')$$

Thus, either equation (5) or (7) can be theoretically justified. Nevertheless, equation (5) introduces additional error beyond that of equation (7) since the error term, u_t, subsumes v_t and w_t. In practice, estimating both forms would allow a cross-checking of results. Whichever reduced form is used, the derivation depends on the small-country assumption. Its validity has rarely, if ever, been examined in an empirical investigation of a bilateral rate. We will return to this point in Section III.

Perhaps, however, the small-country assumption is not taken seriously. In that case, equation (5) is not derived from equations (1)—(4) but from an unspecified model of portfolio selection in both countries. This creates difficulties for interpreting F and F*. Branson and his colleagues use net private international investment positions against the rest of the world to proxy for F and F* because bilateral data is difficult to obtain. They say that it would be better to use bilateral data and assert rather offhandedly that F would equal —F* in the bilateral case: one country's asset is the other's debt. Bilateral data is available for Canada. As long as the model applies only to the private sector, however, F does not equal —F*. Consider the following tables:

Table 1 — *United States' Position against Canada*

A U.S. Government claims on Canada: USCR	C U.S. Government liabilities to Canada: CUSR CUSG
B U.S. private sector claims on Canada: USCG USCP	D U.S. private sector liabilities to Canada: CUSP

Table 2 — *Canadian Position against the United States*

A′ Canadian Government claims on the U.S.: CUSR	C′ Canadian Government liabilities to the U.S.: USCR USCG
B′ Canadian private sector claims on the U.S.: CUSG CUSP	D′ Canadian private sector liabilities to the U.S.: USCP

where USCR are U.S. government holdings of Canadian government assets (reserves); USCG are U.S. private sector holdings of Canadian government assets; USCP are U.S. private sector holdings of Canadian private sector assets; CUSR are Canadian government holdings of U.S. government assets (reserves); CUSG are Canadian private sector holdings of U.S. government assets; and CUSP are Canadian private sector holdings of U.S. private sector assets. Table 1 represents the balance sheet of the United States with respect to Canada and Table 2 the balance sheet of Canada with respect to the United States. The letters A—D and A'—D' refer to the quadrants of each table.

Now if F and F* were the net bilateral asset stocks of the U.S. and Canada and not just of their private sectors, F would equal —F*:

$$[(A + B) - (C + D)] = -[(A' + B') - (C' + D')]$$

Nevertheless, if F and F* are the net private bilateral asset stocks this symmetry breaks down:

$$F = (B - D) \neq -(B' - D') = -F^*$$

In the small country case no inconsistency arises since only one country's net position need be considered; but if the reduced form includes F it should not include F*. Even in this case the use of the net stocks implies that portfolio holders are indifferent between yielding up foreign assets and issuing their own liabilities when settling a debt. This need not be true.

In a two-country case, Branson and his colleagues are simply wrong: F does not equal —F*. If both are included, problems are introduced because even though one is not simply the mirror image of the other, both have the mutual claims of the private sectors as constituent parts, which introduces a strong negative correlation between them.

It seems clear, then, that a model which specifies portfolio selection in two countries must include gross, rather than net asset stocks, in order to avoid the inconsistency associated with net stocks.

Another potential problem is that if the assets and the liabilities of the home country are denominated in different currencies (as is typically the case), then one or the other must be converted at the current exchange rate; and the size of F or F* will depend on the exchange rate that it helps determine. Fortunately for the present investigation, the bulk of Canadian assets held in the United States are U.S. dollar denominated[1]. Thus, re-

[1] Comparisons of the data from the *Survey of Current Business* with data from the *Bank of Canada Review*, statistical Table A.6, published once a year, establish this point.

ducing assets and liabilities to a common, U.S. dollar denominated net asset stock is probably warranted in this case as in few others.

There are then at least three problems with the simple portfolio balance approach as it has been applied: that it is bilateral when a multilateral approach is called for; that it employs the small-country assumption for reasons of mathematical tractability without testing its appropriateness; and that it attempts to collapse all internationally traded assets into a single net asset stock for each country.

III.

There is relatively good bilateral data on the international investment positions of the United States and Canada[1]. The limitations of the available data, however, put the estimation of a multilateral model beyond reasonable hope[2]. Only a bilateral model could be readily estimated in practice because complete data on a third country (representing the rest of the world) cannot be easily obtained.

Nevertheless, the problems of the small-country assumption and of the single internationally traded asset are more easily met: First, in the general case both countries can be modelled explicitly; then the small-country assumption may or may not hold. Second, a separate internationally traded asset can be allowed for each country. The internationally exchangeable asset should be distinguished from the domestic asset in each country because foreign financial assets in private hands consist of claims against the government and against the private sectors of other countries, while only claims against the government of those other countries are assets of their own private sectors; hence, internationally exchangeable assets differ in composition from domestic assets.

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[1] Martin and Masson [1979] do set out a multilateral model, but one in which all but one country are creditors of the one. In these estimations, the United States is the debtor nation and bilateral, net investment positions serve as the one traded asset. This is unsatisfactory for the case of Canada, for instance, because the United States is a net creditor against Canada.

[2] The U.S. Commerce Department's *Survey of Current Business* publishes each year data on the international investment position of the United States with respect to the world as a whole, various regions, Canada and Japan at year's end; and it publishes each quarter the corresponding flow data. Branson *et al.* and Bisignano and Hoover have generated net international positions against the world as a whole by accumulating the current account balance less changes in official reserves on benchmark values of the investment positions. For a bilateral case with gross rather than net asset positions, this would be difficult to do. Fortunately for Canada, one can just as well compute the asset values from capital rather than current account identities, using the data mentioned above. For further information see the appendix.

A sketch of the formal structure of a bilateral model follows:

Country I

$$M = m\ (i, i^*, r, r^*)\ W \quad (8)$$
$$B = b\ (i, i^*, r, r^*)\ W \quad (9)$$
$$-F = f\ (i, i^*, r, r^*)\ W \quad (10)$$
$$e\,(G^* + F^*) = f'\ (i, i^*, r, r^*)\ W \quad (11)$$
$$M + B - F + e\,(G^* + F^*) \equiv W \quad (12)$$

Country II

$$M^* = m^*\ (i, i^*, r, r^*)\ W^* \quad (13)$$
$$B^* = b^*\ (i, i^*, r, r^*)\ W^* \quad (14)$$
$$-F^* = f^{*\prime}\ (i, i^*, r, r^*)\ W^* \quad (15)$$
$$(G + F)/e = f^*\ (i, i^*, r, r^*)\ W^* \quad (16)$$
$$M^* + B^* + (G+F)/e - F^* \equiv W^* \quad (17)$$

$$f'\ W/(G^* + F^*) \equiv (G + F)/f^*\ W^* \quad (18)$$

where M, B, W, m, b, e and i are defined as before; F now represents the foreign liabilities of the private sector of Country I; G represents the foreign liabilities of the government in Country I *held by the private sector* of Country II; and r is the rate of interest on F; f and f' are the desired proportions of wealth held respectively as liabilities and as assets against the foreign country. Starred variables are the corresponding variables for Country II.

The system of equations (8)—(18) has eight behavioral equations and three identities (The identity, equation (18), follows from the fact that the same exchange rate e appears in equations (11) and (16).) Thus, five independent equations determine four interest rates and one exchange rate. The variables G and G* are necessary because, as described in Section II, government assets held in the private sector of foreign countries are assets of that sector, but they are not liabilities of the *private* sector of the home country.

The system above can be solved recursively: solving equations (8), (9), (13) and (14) yields the four interest rates; then by substituting these into the remaining equations and using the three identities, the exchange rate can be determined. In the general case, the reduced form for the exchange rate will be:

$$e_t = \xi\ (M_t, B_t, F_t, G_t, M_t^*, B_t^*, F_t^*, G_t^*) + u_t \quad (19)$$

Equation (19) has the advantage over the usual reduced form, equation (5), of including both countries' internationally exchangeable asset stocks as a consequence of the underlying model and not as an ad hoc addition to it. Furthermore, the inclusion of G and G* permits accounting identities to be respected.

In spite of its advantages, which are principally a consequence of greater generality, the model in equation (8) to (18) has a great drawback —

namely, that it does not permit one to assign a priori the sign of the effect of changes in the asset stocks on the exchange rate in the reduced form. On deriving the comparative statics by standard methods, it quickly becomes apparent that, in general, the direction of the effect of a change in any of the asset stocks depends on the values of the functions m, b, f, f' and their starred counterparts for any given configuration of the interest rates and the exchange rate. Of course, this may well be the truth about economic reality. Still, it would be altogether simpler econometrically if, in fact, restrictive assumptions held which would permit the general model to be reduced to a simpler one more like that of equations (1)—(4).

The key to simplifying the model, though, is establishing the small country assumption. If the U.S. interest rate is exogenous, then one is justified in paying attention to the Canadian side as in the simpler model, rather than to both sides as in the more general model.

A way of testing exogeneity is to use tests of *Granger-causality*. A variable X *Granger-causes* Y if the past values of X and of Y better explain Y than past values of Y alone. Hence, in Granger's test if Y is regressed on lagged values of Y and of X and if all the coefficients on X are insignificant, then the null hypothesis that Y does not cause X cannot be rejected[1]. Sims [1972] has shown that Granger-causality is econometrically equivalent to exogeneity.

The program for establishing the small-country assumption is thus clear. The Granger test should be applied to U.S. interest rates and Canadian assets in both directions. For the small-country assumption to hold, Canadian assets must not Granger-cause U.S. interest rates, but U.S. rates should Granger-cause Canadian assets. Since U.S. rates must be independent both of changes in Canadian assets and liabilities in the aggregate and of changes in their composition, the individual assets M, B, F, F*, G and G* as well as the aggregate W should be tested for Granger-causality.

IV.

The considerations in the preceding sections suggest the following program of empirical investigation:

(a) the reduced form, equation (19), can be estimated;

(b) the tests for Granger-causality discussed in Section III can be run;

(c) if (b) produces the results necessary to establish the small-country assumption, then a reduced form

[1] For an excellent review of the application of various tests, particularly Granger's own test, see Feige and Pearce [1979].

$$e_t = \gamma\,(M_t, B_t, F_t, G_t, F_t^*, i_t^*) + z_t \qquad (20)$$

where z_t is an error term, can be estimated;

(d) again contingent on the results of (b), a reduced form

$$e_t = \lambda\,(M_t, B_t, Net_t, i^*) + x_t \qquad (21)$$

where x_t is an error term and Net $= (F + G) - F^*$, can be estimated.

We noted in Section II that Branson *et al.* as well as Bisignano and Hoover [1980, Table 12] in their earlier work do not estimate the model set out in equations (1)—(4), which implies a reduced form like that of equation (7). Therefore, before presenting the results of the program just described, it might serve as a useful benchmark to estimate the reduced form

$$e_t = \psi\,(M_t, B_t, F_t, i_t^*) + v_t \qquad (7)$$

where the unstarred independent variables are Canadian asset stocks and the starred interest rate i^* is the U.S. short rate. The dependent variable, e, is the U.S. dollar/Canadian dollar exchange rate.

The results of these estimates are shown in Table 3. The first two lines are ordinary least squares (OLSQ) and the OLSQ with Cochrane-Orcutt correction for serial correlation (CORC) estimates using the net international investment position of the Canadian private sector against the rest of the world. Line 3 presents the FAIR technique estimates using the same investment position. (The FAIR technique is a Cochrane-Orcutt type correction which adjusts for the presence of endogenous variables by means of instrumental variables [Fair, 1970]; in this case, Canadian reserve money is assumed to be endogenous because of government intervention in foreign exchange markets.) This is the cumulation of the current account less changes in reserves over the benchmark estimates [Bisignano and Hoover, 1980, Appendix, pp. III, IV, XI, XII]. Lines 4—6 present OLSQ, CORC and FAIR technique estimates of the same reduced form but with a net bilateral investment position for the Canadian private sector against the United States[1].

The results in Table 3 are similar to the earlier results reported by Bisignano and Hoover. All the variables have the correct sign except reserve money in line 1 and the foreign debt stock in lines 4—6. (This assumes that the "correct" sign on the Canadian domestic bond stock is positive. To sign this coefficient requires a prior knowledge of the

[1] The net bilateral investment position (B′—D′ in Table 2) equals CFM + CGM — USFM. For definitions, sources and methods of calculating these variables see the appendix.

Table 3 — *Portfolio Balance Model of U.S. Dollar/Canadian Dollar Exchange Rate*, 1973.03—1978.12

Estimation method	Independent variables[a]					Summary statistics			
	constant	$-RM_C$	$+D_C$	$+F_C$[b]	$-r_{US}$	rho	$\bar{R}^2$	S.E.R.	D.W.
(1) OLSQ	1.050 (31.4)	.0134 (2.12)	.0170 (5.72)	.107 (10^{-4}) (6.05)	—.0080 (6.64)	—	.874	.194	.56
(2) CORC	1.205 (14.36)	—.0140 (1.11)	.0060 (2.55)	.434 (10^{-5}) (1.38)	—.0018 (.93)	.90	.957	.0113	2.26
(3) FAIR[c]	1.204 (.1131)	—.0195 (1.09)	.0060 (2.53)	.318 (10^{-5}) (.76)	—.0016 (.84)	.90	.957	.0113	2.26
(4) OLSQ	1.184 (39.65)	—.0277 (9.29)	.0171 (4.66)	—.187 (10^{-5}) (1.37)	—.0074 (4.59)	—	.808	.0239	.38
(5) CORC	1.261 (16.95)	—.0312 (4.74)	.0060 (2.49)	—.968 (10^{-6}) (.80)	—.0013 (.67)	.90	.957	.0114	2.22
(6) FAIR[c]	1.244 (16.79)	—.0342 (4.93)	.0059 (2.47)	—.118 (10^{-5}) (.94)	—.0012 (.63)	.89	.957	.0113	2.23

[a] RM = reserve money; D = domestic debt held by domestic residents; F = net international investment position of the private sector against the rest of the world; F^P = domestic private holdings of foreign private debt; F^G = domestic private holdings of foreign government debt; r = short-term rate of interest; C = Canada; US = United States. — [b] In lines 1—3 only F, in lines 4—6 net new = $F^P_C + F^G_C - F^P_{US}$ (for calculation see the appendix). — [c] RM_C assumed to be endogenous. — t-statistics in parentheses.

relative substitutability of domestic and foreign bonds and domestic bonds and money, with the former being larger.) In the first case, the Durbin-Watson test statistic indicates significant first order serial correlation. When this is corrected for in lines 2 and 3, reserve money carries the proper sign. The foreign debt stock in lines 4—6 carries the wrong sign; however, it is insignificant at the 95 percent level in all three cases.

Only the domestic debt stock is consistently significant. The foreign debt stock is significant in line 1, but the bilateral stock is not significant in line 5. With either foreign debt stock, serial correlation wipes out the significant t-statistics. Reserve money is very sensitive to estimation technique when the wider foreign asset stock is entered. It is significant when OLSQ is used in line 1 but loses significance with CORC and FAIR in lines 2 and 3. In contrast, when the narrower foreign debt stock is used, reserve money is significant regardless of the estimation technique. So far these results are similar to our earlier estimations. The goodness of fit, $\bar{R}^2$, is similar in both sets of estimates. Surprisingly, the U.S. short rate is insignificant in all but the badly serially correlated estimate in lines 1 and 4. Still it bears the proper sign in every case.

Let us now consider the estimation of the reduced form, equation (19). Estimates are presented using OLSQ, CORC and FAIR techniques in Table 4. As is well known in the presence of significant serial correlation the estimated coefficients' standard errors are biased downward. Given this caveat, equation (19) estimated with OLSQ (column 1) seems to imply that the disaggregation of the foreign asset stocks was worthwhile, indicating that the exchange rate is influenced by the private holdings of foreign private debt. The importance of domestic private holdings of foreign private debt is additionally confirmed in columns 2 and 3. As with our earlier results, an increase in U.S. private holdings of Canadian private liabilities depreciates the U.S. dollar. None of the other foreign asset stocks is significant once serially correlation corrections are made (columns 1 and 2).

The signs of the two reserve money variables are the same as in our earlier results. An expansion of the U.S. monetary base depreciates the U.S. dollar, while an expansion of the Canadian monetary base depreciates the Canadian dollar. The coefficients are insignificant on the U.S. variable, but significant on the Canadian variable except where estimated with OLSQ. Canadian domestic debt stocks, which were found to have rather consistent appreciatory effects on the Canadian dollar in our earlier results (and in Table 3) are found in Table 4 to be significant only in column 1 estimated by OLSQ.

The results for the reduced form, equation (19), as presented below, are rather mixed and difficult to assess because the coefficients cannot be signed a priori unless the small-country assumption holds. Table 5 presents the results of the Granger-causality tests described in Section III by which the small-country assumption may be tested. The tests are run between Canadian asset stocks and the U.S. short rate in both directions. Canadian assets include money, domestic bonds, foreign assets (holdings of U.S. private assets with and without direct investment, holdings of U.S. governments assets, and the net foreign investment position) and net private wealth (money + domestic bonds + net bilateral investment position).

The test consists of a regression of the presumed *Granger-caused* variable on lagged values of itself and on lagged values of the presumed *Granger-causing* variable. Third degree, polynomial distributed lags of 12 months on the dependent variable and 16 months on the independent variable are estimated in every case. White residuals are obtained by prefiltering the variables with a third or eighth order autoregression correction[1]. Then F-statistics are calculated to test the group significance of the presumed Granger-causing variables.

In all the lines but 3, 4, 9—11, 13 and 14 the F-statistics are below the critical values at the 95 percent confidence level, indicating that the

[1] But see Table 5, footnote e.

Table 4 — *Portfolio Balance Model of U.S. Dollar/Canadian Dollar Exchange Rate,* 1973.03—1978.12

	Estimation method		
	OLSQ	CORC	FAIR
	1	2	3
Independent variables[a]			
constant	1.125 (14.77)	1.193 (13.88)	1.218 (14.14)
$^{+}RM_{US}$	.64 (10^{-3}) (.52)	.644 (10^{-3}) (.86)	.979 (10^{-3}) (1.28)
$^{-}RM_{C}$	—.0056 (.46)	—.0317 (2.12)	—.0537 (2.67)
$^{-}D_{US}$	.56 (10^{-4}) (.28)	—.456 (10^{-3}) (.16)	.826 (10^{-4}) (.21)
$^{+}D_{C}$	—.015 (6.73)	—.224 (10^{-3}) (.61)	.55 (10^{-3}) (.19)
$^{-}F^{P}_{US}$	.433 (10^{-5}) (3.42)	.394 (10^{-5}) (2.35)	.456 (10^{-5}) (2.72)
$^{+}F^{P}_{C}$	.420 (10^{-5}) (2.91)	.916 (10^{-6}) (.51)	.353 (10^{-6}) (.20)
$^{-}F^{G}_{US}$	.106 (10^{-5}) (.07)	—.360 (10^{-4}) (1.54)	—.269 (10^{-4}) (1.16)
$^{+}F^{G}_{C}$	.315 (10^{-3}) (1.35)	.616 (10^{-5}) (.02)	.145 (10^{-3}) (.41)
Summary statistics			
rho	—	.86	.84
$\bar{R}^2$	.909	.955	.957
S.E.R.	.0164	.0116	.0114
D.W.	.85	2.24	2.25

[a] "Net new" data as defined in Table 3, footnote b. For variable definitions see Table 3. — t-statistics in parentheses.

null hypothesis that the independent variable does not Granger-cause the dependent variable cannot be rejected. This suggests that the Canadian assets in these lines are independent of U.S. interest rates and vice versa.

The remaining seven lines contain three pairs (3, 4; 9, 10; 13, 14) for which the null hypothesis is rejected at the 95 percent level, suggesting that Canadian reserve money, holdings of U.S. government assets and net private wealth do Granger-cause U.S. interest rates and vice versa. Mutual causation tells against the small-country assumption: U.S. rates are not exogenous if they are Granger-caused by any Canadian assets. Only in the case of lines 11 and 12 does a pattern supporting the small-country assumption appear: Canadian net private foreign asset position is apparently Granger-caused by U.S. interest rates (line 11) but does not Granger-cause them (line 12).

Some care must be taken in assessing these results. Every regression of a Canadian asset on U.S. interest rates in Table 5 that rejects the small-

Table 5 — *Tests of Granger-Causality: The U.S. Short-Term Interest Rate and Canadian Asset Stocks,* 1972.01 — 1980.06

	Regression[a]	Order of auto-correlation correction[b]	Degrees of freedom[c]	F-statistic[d]
(1)	Canadian domestic bonds on U.S. short rate	3	(3,83)	1.91
(2)	U.S. short rate on Canadian domestic bonds	3	(3,83)	2.04
(3)	Canadian reserve money on U.S. short rate[e]	3	(3,78)	3.85
(4)	U.S. short rate on Canadian reserve money	3	(3,83)	3.66
(5)	Canadian private holdings of U.S. private assets on U.S. short rate	8	(3,78)	1.55
(6)	U.S. short rate on Canadian private holdings of U.S. private assets	3	(3,83)	2.02
(7)	Canadian private holdings of U.S. private assets less direct investment on U.S. short rate	8	(3,78)	2.02
(8)	U.S. short rate on Canadian private holdings of U.S. private assets less direct investment	3	(3,83)	2.08
(9)	Canadian private holdings of U.S. government assets on U.S. short rate[e]	8	(3,78)	3.58
(10)	U.S. short rate on Canadian holdings of U.S. government assets	3	(3,83)	3.89
(11)	Canadian net private foreign asset position against the U.S. on U.S. short rate[e]	8	(3,78)	2.86
(12)	U.S. short rate on Canadian net private foreign asset position against the U.S.	3	(3,83)	1.90
(13)	Canadian private net wealth on U.S. short rate[e]	8	(3,78)	3.79
(14)	U.S. short rate on Canadian private net wealth	3	(3,83)	3.65

[a] In each regression, the dependent variable is regressed on lagged values of itself and of the independent variable. The null hypothesis is that the independent variable does not Granger-cause the dependent variable — i.e., that the coefficients on the lagged independent variable are insignificant as a group. Each regression (Almon technique) includes a constant and third degree, polynomial distributed lags, 12 observations long on the dependent variable and 16 observations long on the independent variable. — [b] In order to secure "white" residuals required for the Granger test, the data was pre-filtered by autoregressive transformations of the Cochrane-Orcutt type with autoregressive terms of the 1st through nth order, where n is the number in this column. — [c] The first number is the number of degrees of freedom in the numerator of the F-statistic; the second, the number in the denominator. — [d] The proper F-statistic is as follows: $F = [(\Sigma e^{*2} - \Sigma e^2)/R]/[\Sigma e^2/(N-K-1)]$ where e = residuals from the unrestricted regression; e* = residuals from the restricted regression; R = number of restrictions; N = number of observations; and K = variables in the unrestricted regression. Because one scrambled variable is generated for each degree of the polynomial specified for each variable in the Almon technique, R = 3 and K = 6. The critical value $F_{.95}$ (3,78) = 2.73; $F_{.95}$ (3,83) = 2.72; $F_{.99}$ (3,78) = 4.06; $F_{.99}$ (3,83) = 4.04. F-statistics greater than the appropriate critical value, which therefore reject the null hypothesis at the 95 percent confidence level, are underlined. — [e] The residuals of the unrestricted or the restricted regression are not "white."

country hypothesis also has non-white residuals in the restricted or unrestricted form. This is contrary to the theoretical requirements of the Granger test. More sophisticated pre-filtering might well secure the required white residuals. Nevertheless, there is reason to believe that the value of the F-statistic may be fairly insensitive to differences in pre-filtering[1].

On the basis of these preliminary results, the small-country assumption should probably be rejected. There seems to be Granger-causality between

Table 6 — *Portfolio Balance Model of U.S. Dollar/Canadian Dollar Exchange Rate,* 1973.03—1978.12

	Estimation method[a]	
	FAIR for (20)	FAIR for (21)
	1	2
Independent variables[b]		
constant	1.089	1.244
	(17.77)	(16.79)
$^{-}RM_C$	—.042	—.0342
	(2.52)	(4.93)
$^{+}D^C$	—.0019	.0059
	(.94)	(2.47)
$^{-}F^P_{US}$	.627 (10^{-5})	—
	(1.67)	
$^{+}F^{P\,c}_C$	.203 (10^{-5})	—.118 (10^{-5})
	(1.32)	(.94)
$^{+}F^G_C$	—.334 (10^{-3})	—
	(1.21)	
$^{-}r_{US}$	—.0035	—.0012
	(3.27)	(.68)
Summary statistics		
rho	.89	.89
$\bar{R}^2$	.972	.957
S.E.R.	.0111	.0113
D.W.	2.05	2.23

[a] RM_C assumed to be endogenous. — [b] For variable definitions see Table 3. — [c] In the equation in column (2) the net Canadian foreign asset stock is defined as the net stock = $F^P_C + F^G_C - F^P_{US}$.

[1] Geweke [1978] gives an example illustrating the relative insensitivity of the F-statistic to a range of pre-filters.

some Canadian assets and U.S. interest rates; U.S. rates, therefore, are not exogenous with respect to the Canadian portfolio.

Strictly speaking, the results just presented remove the reason for estimating equations (20) and (21). Nevertheless, as the results are *preliminary*, Table 6 presents these estimates anyway—column 1 is a FAIR estimate of (20) and column 2 a FAIR estimate of (21). In column 1, Canadian reserve money, U.S. private holdings of Canadian private assets (i.e., Canadian private liabilities to the U.S. private sector) and the U.S. interest rate are significant at the 95 percent confidence level. In column 2, the three foreign asset stocks are aggregated. This seems to raise the significance of Canadian domestic bond stock, but also to lower the significance of the U.S. interest rate. The stock itself is insignificant. Aggregation of the foreign asset stock also reduces the overall explanatory power of the equation.

V.

The simple portfolio model proposed by Branson *et al.* [1977] has been used by several investigators to illustrate the role of stocks of both monetary and non-monetary, interest-bearing assets in the determination of the exchange rate. These models are largely in the spirit of portfolio balance models associated with the work of James Tobin. Both theoretical and empirical ambiguities have plagued these models, partly resulting from the theoretical specification of a small-country, bilateral exchange rate model but the estimation of models which often bear only modest resemblances to the theoretical specification. Three reasons seem to be common: the theoretical convenience of the small-country assumption; the lack of reliable international bilateral asset stock data; and the lack of consistency in the use of inside-outside money and financial asset distinctions. This paper has sought to address some of these issues and has met with modest success. It is not at all clear, for example, that the often used small-country assumption used for large financially developed economies can be justified empirically. Secondly, a bilateral exchange rate model should use bilateral financial data to the extent possible. The modest success the authors have met with portfolio models of the Branson *et al.* variety also suggests that exchange rate investigators have neglected the role of domestic non-monetary asset stocks in exchange rate determination.

Data Appendix

1. Source of Data

BCR *Bank of Canada Review*, Ottawa, various issues

IFS International Monetary Fund, *International Financial Statistics*, and associated data tape, Washington, December 1980

SCB U.S. Department of Commerce, Bureau of Economic Analysis, *Survey of Current Business*, Washington, various issues

2. Series Definitions

Country prefixes: C = Canada; US = United States
Series names, definitions and units (in parentheses):

DM direct investment overseas: see below "Calculation of Bilateral Asset Stocks" (mill. U.S. $)

FM private holdings of foreign private assets: see *ibid.* (mill. U.S. $)

GM private holdings of foreign government assets: see *ibid.* (mill. U.S. $)

RM official international reserves: see *ibid.* (billion U.S. $)

RMM reserve money (monetary base): see *IFS*, line 14 (billion C $ and U.S. $)

Adjustment: Unless otherwise noted, all series are available from their source monthly. Quarterly series are interpolated to monthly (see "Interpolation", below). All series, except exchange rates and interest rates, which are seasonally adjusted at their source, are seasonally adjusted using the X11 method (multiplicative version). Components are adjusted before calculation.

3. Calculation of Bilateral Asset Stocks

The general method of calculating bilateral asset stocks was the same for each of the eight series — CDM, CFM, CGM, CRM, USDM, USFM, USGM and USRM: Data from "International Investment Position of the United States", *SCB*, serve as benchmarks for each of the series above for the end of the years 1971—1979. The first differences of these series are the changes over the course of a year. Equivalent flow components for each series from quarterly tables in the *SCB* were then used to distribute the year over year change cumulatively over the four quarters in proportion to the quarterly flows. Ideally, the quarterly flows should sum to the year over year change, but, because of errors and omissions, they do not always. The method described distributes the errors and omissions over the four quarters in proportion to the ratio of the quarterly flow to the sum of the four quarterly flows in the year. For the two quarters in 1980, the flows are added directly. Once quarterly stocks have been generated, monthly ones are generated by interpolating (see "Interpolation", below).

Formally: If $B_{i,4}$ is the benchmark value of a series at the end of year i, then: $\Delta B_i = B_{i,4} - B_{i-1,4}$

Then, if $F_{i,j}$ is the flow counterpart to B in the j^{th} quarter of the i^{th} year, $B_{i,j} = (\sum_{k=1}^{j} F_{i,k}) \cdot \Delta B_i + B_{i-1,4}$.

The tables below indicate the precise sources of data for the benchmarks and flows.

Source for Benchmarks:

"International Investment Position of the United States", Table 3, *SCB*, August 1973; 1974; 1976; 1978; 1979 and October 1975; 1977; 1980.

Series / Year	CDM	CFM	CGM	CRM	USDM	USFM	USGM	USRM
	(Table 3 line numbers)							
1973—1975	30	28 + 35 + 37	28	41	9	8 + 14 + 18[a]	[a]	4 + 21
1976—1980	33	32	29	25—29	14	13[a]	[a]	3 + 8

[a] Lines 8 + 14 + 18 up to 1975 and line 13 thereafter contain both USFM and USGM. In order to apportion the assets between these two categories, USGM is taken to be equal to the U.S. dollar denominated share of government of Canada bonds. Then USFM = (8 + 14 + 18) — USGM. The source for USGM is *BCR*, August 1972, September 1973—1978, and October 1979, Table A.6, line "Government of Canada, Direct + Guaranteed", column "U.S. dollars". These figures were converted to U.S. dollars using *IFS*, line ag (annual). USGM for 1979 was defined as: $[USGM_{1978}/(13)_{1978}] \cdot (13)_{1979}$.

Source for Flows:

"U.S. International Transactions," *SCB*, December 1973—1979.

Year	Table	Series							
		CDM	CFM	CGM	CRM	USDM	USFM	USGM	USRM
		(line numbers of the respective table)							
1973—1975	9	49	49 + 50 + 51 + 52	48	(53 to 56) + 57	39	48[a]	[a]	33
1976—1977	10	59	58	55	51—55	44	43[a]	[a]	34 + 39
1978—1980	10	65	64	61	57—61	48	47[a]	[a]	38 + 43

[a] Lines 48 up to 1975, 43 from 1976 to 1977 and 47 thereafter contain both USFM and USG. In order to apportion the flows between these two categories, the quarterly figures for any year are multiplied by the ratio of the benchmarks USGM and USFM to the sum (USGM + USFM) for the corresponding year. For the years 1979—1980, the same ratio is used for 1978.

4. Interpolation

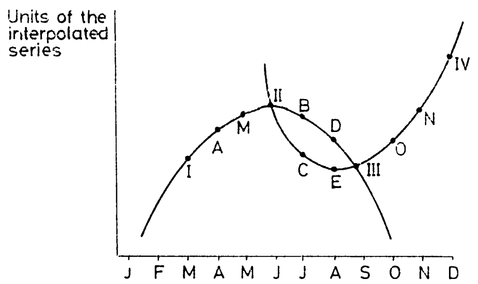
The interpolation scheme used in transforming quarterly series into monthly series works as follows:

(1) A parabola is fitted to the first three quarters, another to quarters two to four, another to three to five and so forth, using the quarterly value as the last month in the quarter.

(2) From (1) there corresponds to the region between the first and second quarters and to that between the penultimate and final quarters a segment of a parabola. Monthly values are taken from these segments.

(3) From (1) there corresponds to every region not covered in (2) segments of two different parabolas. Monthly values are the arithmetic mean of values taken from these segments.

To illustrate this graphically:



The value for: March = I; April = A; May = M;
June = II; July = (B + C)/2; August = (D + E/2;
September = III; October = O; November = N;
December = IV.

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* * *

Zusammenfassung: Einige Verbesserungen eines einfachen Modells des Vermögensansatzes zur Wechselkursbestimmung unter besonderer Berücksichtigung des Wechselkurses zwischen dem amerikanischen und kanadischen Dollar. — Einfache Modelle des Vermögensansatzes zur Wechselkursbestimmung enthalten formal nur ein Land mit drei Arten von Vermögensanlagen: zinsloses Geld, ein verzinstes heimisches Wertpapier und ein verzinstes international gehandeltes Wertpapier. Diese Modelle sind nur unter sehr restriktiven Annahmen plausibel. Es ist der Zweck dieses Aufsatzes, die Struktur eines solchen Modells und die ihm zugrunde liegenden Annahmen darzustellen. Bei der einfachen Version dieses Ansatzes gibt es, so wie er benutzt wurde, mindestens drei Probleme: er ist bilateral, obwohl ein multilateraler Ansatz erforderlich wäre; er unterstellt wegen der leichteren mathematischen Handhabbarkeit den Fall eines »kleinen Landes«, ohne die Zweckmäßigkeit dieser Annahme zu prüfen; er versucht für jedes Land, alle international gehandelten Wertpapiere in einem einzigen Netto-Wertpapierbestand zusammenzufassen. Nach der Untersuchung dieser Annahmen wird ein allgemeines Modell vorgeschlagen, das im wesentlichen einer solchen Kritik nicht ausgesetzt ist, und schließlich werden im Hinblick auf den Wechselkurs zwischen dem amerikanischen und kanadischen Dollar einige empirische Ergebnisse vorgelegt.

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Résumé: Quelques améliorations proposées d'un simple modèle de balance de portefeuille de la détermination du taux de change avec référence particulière au taux de dollar des E.U./dollar canadien. — Les simples modèles de balance de portefeuille des taux de change formalement n'incluent qu'un seul pays avec trois actifs: la monnaie pas portant des intérêts, un actif local portant des intérêts et un actif internationalement commerçable qui porte des intérêts. Ces modèles ne sont plausibles que sous des suppositions très restrictives. Le but de cet article est d'éclairer la structure et les suppositions d'un tel modèle. Il y a au moins trois problèmes avec la simple approche appliquée: elle est bilaterale quoique une approche multilaterale soit exigée; elle emploie la supposition de petit pays à cause de la traitabilité mathématique sans tester si elle est justifiée et elle essaie d'agréger tous les actifs internationalement commercés dans un seul stock d'actif net pour chaque pays. Puis les auteurs proposent un modèle plus général qui est largement exempt de ces critiques et, finalement, ils présentent quelques résultats empiriques pour le taux du dollar des E.U./dollar canadien.

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Resumen: Algunos mejoramientos sugeridos para un modelo de cartera de equilibrio simple para la determinación del tipo de cambio con especial referencia a la relación cambiaria U.S. dólar/dólar canadiense. — Modelos de cartera de equilibrio simple para la determinación de tipos de cambio formalmente involucran a un solo país con tres activos: dinero no portador de intereses, un activo doméstico portador de intereses y un activo portador de intereses transable internacionalmente. Estos modelos son solamente plausibles bajo supuestos muy restrictivos. El propósito de este artículo es esbozar la estructura y supuestos de un modelo de este tipo. Hay por lo menos tres problemas con el planteamiento de cartera de equilibrio simple como se ha aplicado: que es bilateral cuando se necesita una aproximación multilateral; que emplea el supuesto de un país pequeño por razones de manejo matemático sin comprobar si es apropiado; y que pretende desplomar todos los activos transados internacionalmente dentro de un stock de activos neto único para cada país. En seguida se propone un modelo más general que está en su mayor libre de estas críticas y finalmente, con referencia a la relación cambiaria U.S. dólar/dólar canadiense, se presentan algunos resultados empíricos.