

# Evaluating A Buy and Hold Strategy for the S&P 500 Index

*Edward Tower & Omer Gokcekus\**

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## **Abstract**

In this paper, we calculate the real rate of return from purchasing the S&P 500 index from 1871 through 2001. We assume the investor purchases the index in January of each year and holds it forever, consuming dividends, but never selling the index itself or else selling it after its present value is dwarfed by the present value of the dividend flows. The calculations rest on best guesses about post 2000 dividends. These we infer from past behavior. The highest real return was 13.02% for a purchase in June 1932. The lowest was 2.88% for August 2000. The expected return for a purchase in January 2001 was 3.08%. To raise it to the 5% that we judge the minimum return necessary to maintain investor interest, the S&P 500 index would have had to fall by 53% from its January 2001 value of 1336 to 624.

\* Edward Tower, [tower@econ.duke.edu](mailto:tower@econ.duke.edu), Professor of Economics, Department of Economics, Duke University, Omer Gokcekus, [ogokceku@wpo.nccu.edu](mailto:ogokceku@wpo.nccu.edu), Assistant Professor of Economics, School of Business, North Carolina Central University.

For Correspondence: Edward Tower, Department of Economics, Box 9007, Duke University, Durham, NC 27708-0097; telephone: 919-660-1818; fax: 919-684-8974.

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# Evaluating A Buy and Hold Strategy for the S&P 500 Index

## I. Introduction

Just after the publication of Robert Shiller's *Irrational Exuberance* in March 2000, Ed Tower tried to convince his sister to dump stock in her retirement account and buy bonds. After reflection, she said that she didn't care about the crash he predicted, since she is a long-term investor.

Shiller's statistical argument that the stock market was overvalued in 2000 rests primarily on a simple graph. He calculates PE10, the ratio of the real price of the S&P 500 index to the average of the index's real earnings over the previous 10 years. He then uses PE10 to predict the 10-year real rate of return (including dividends) from purchasing the index (and selling it 10 years later). He finds a negative correlation: A high PE10 predicts a low real rate of return. Since capital gains play an important role in these calculations, the argument hinges in part on investors' tendency to beat the price back to its normal multiple of a ten-year average of earnings.

From time to time investors may change the real rate of return they require to maintain interest in the stock market as interest rates, brokerage costs, mutual fund expense ratios, tax laws, the attractiveness of alternative investments and attitudes all change. So the question arises, can we explore the issue of whether the stock market has long term appeal in a way that does not hinge on investor behavior? If we were to calculate the real rate of return (henceforth just return) assuming dividends are reinvested we need to make

some assumption about future prices at which the index can be purchased with these dividends.

Moreover there is another problem with calculating the real rate of return from the stock market when dividends are continually reinvested, even if we assume that the index is sold only after a very long time. If we assume that after some point in the future a steady state emerges in which real dividends and real prices both rise at a proportional rate of  $g$  per year, with a dividend yield (the ratio of real dividends to real stock price) of  $y$ , then the return from the stock market in the limit as the steady state period approaches forever, will be  $g + y$  per year, regardless of at what price the index is purchased.<sup>1</sup> So, ironically, the purchase price does not matter.

To eliminate both these problems we calculate the return on a hypothetical stock market investment, assuming dividends are consumed rather than reinvested. To accomplish this, we assume that beginning in January 2001, when our data ends, real dividends rise steadily at a constant proportional rate,  $g$ , and that the real price of the index rises no faster than  $g + y_{2001}$ .

The real rate of return on such an investment after January 2001 must exceed  $g$  (by  $y$ ). Thus if we consider a sufficiently long time horizon, the present value of the sales price of the index, calculated using this rate of return, must equal zero. Consequently, we can calculate the return on the investment, based on dividends only, and not worry about how the animal spirits of bulls and bears bounce the stock price around.<sup>2</sup>

Our hope is to give Ed's sister and those like her a useful tool for evaluating the stock market. In the process we draw some conclusions about whether the price earnings ratio is a good predictor of the stock market's rate of return and which time lags are best in calculating the ratio.

## **II. Robert Shiller's Data**

This paper explores approaches which draw heavily on ideas from Shiller in his two books, [1989] and [2000], and his two articles with John Campbell [1998] and [2001].

All of our data is drawn solely from Shiller, which he graciously and conveniently provides on his web site (<http://www.econ.yale.edu/~shiller/>).

He provides monthly data on price, dividends and earnings for the S&P 500 and the consumer price index from 1871, shortly after the Civil War ended in 1865, through 2000. His figures for the price of the S&P 500 are monthly averages of daily closing prices. In all of our calculations, we use his monthly values for all variables, unless noted otherwise. Dividing nominal values by the CPI yields real values. Henceforth all values are real including rates of return, and generally we drop the term real. This data is easily downloadable into Microsoft Excel, so it is easy for the reader to check and evaluate the work in this paper.

In reading the data, we were struck by how non-monotonic it is. Looking at Shiller's monthly averages: (A) real price for the S&P index reached a peak in 1906, which was

not reached again until 1928, and it did not remain permanently higher until 1982; (B) annual real earnings reached a peak in 1916, which was not reached again until 1955; (C) annual real dividends reached a peak in 1966, which was not reached again until 1990, and did not remain permanently higher until 1995. These points are illustrated in Figure 1, which graphs the natural logs of monthly history of the S&P 500's real price, real earnings and real dividends using monthly data from January 1871 through the latest dates available in 2001.

Figure 2 graphs the cumulative real return from investing in the S&P 500 index assuming the reinvestment of dividends. It is useful for seeing how long it has taken investors to recover from down markets.<sup>3</sup> In real terms, an August 1902 investment was still a loser in April 1921. A September 1929 investment was still a loser in March 1948. A March 1961 investment was still a loser in November 1974.

Figure 2 also graphs the cumulative return on an investment in the S&P 500 financed by borrowing on margin at a 3% annualized real interest rate. Using this more stringent test of performance we find several long periods of negative returns: October 1886 to August 1921, August 1898 to June 1932, July 1926 to April 1943, June 1955 to July 1982 and December 1968 to October 1990. Another way to think about this calculation is that it identifies the periods in which the stock market performed less well than the ten-year Treasury inflation protected securities (TIPS) available with a 3% yield in November 2001.

In both figures, each horizontal grid represents a doubling of value. Thus, the stock market is risky and it appears that a long series of current and past values for any of these series is needed to predict future values, a hypothesis that is confirmed by regression analysis. In sections III-V we try to figure out how long these lags are.

### III. What Earnings Period Do Investors Look at?

Shiller [1990] argues that investors are myopic, tending to overreact to recent disturbances. Bernstein [2001] refers the lack of perspective that comes from failing to pay due heed to the past as “recency.” Let’s test Shiller’s hypothesis.

To find out what time horizon investors consider in valuing the stock market, we regressed the natural log of real price in January,  $P$ , on the natural log of the previous years’ average real earnings, using moving averages over various time periods (1 year, 2 years, etc.) to calculate real earnings. We discover that one-year real earnings,  $E1$ , is the best predictor of real price,  $P$ .<sup>4 5</sup> Here is the winning regression using Shiller’s observations on real price for Januarys from 1871 to 2001, with numbers in parentheses denoting  $t$  values.

$$\ln P = 0.991 \ln E1 + 2.65 \quad R^2 = 0.773 \quad (1)$$

(21.0)     (21.1)

A one percent increase in real earnings increases real price by almost one percent.

#### IV. Predicting the Short and Intermediate Run Performance of the Stock Market

As previously mentioned, Shiller uses PE10, the denominator of which is a 10-year moving average of real earnings to predict the 10-year return. He does not defend this choice except by noting Graham and Dodd's [1934] use of it and that 10 years smoothes out events such as earnings fluctuations due to wars and business cycles. Let's figure out the best time lag by using variants of Shiller's calculations to predict one-year real rates of return.

Our goal is to say something useful about predicting rates of return. Jeremy Siegel [1998] presents some evidence that the earnings yield is a decent predictor of the real rate of return for the U.S. stock market. This makes the earnings yield, which is the reciprocal of the price earnings ratio, an interesting variable to work with. We wish to explore Siegel's idea further. So, henceforth, we work with the earnings yield, rather than the PE ratio.

We regress the instantaneous average percentage real rate of return over one year (assuming dividends are reinvested in the same month that they are disbursed and the index is sold at the end of the year) on earnings yields (expressed as a percent of Shiller's current real price), using moving averages over alternative lags to calculate average earnings. We find an 11-year average gives the highest  $R^2$ . Using data on returns between Januarys from 1881-2001 that regression is:

$$\begin{array}{rcll} \text{Return1} = 1.020 \text{ EY11} & -0.813 & R^2 = 0.0295 & (2) \\ & (1.89) & (-0.191) & \end{array}$$

Each 1% increase in the eleven-year moving average of real earnings to current price, EY11, which we name the 11-year earnings yield, raises the instantaneous real rate of return over one year,  $return_1$ , by 1.02 percentage points.

When we redo the analysis to predict the 10-year instantaneous average percentage real rate of return, we discover that the earnings yield (expressed as a percent) which maximizes  $R^2$  is calculated using a lag of 51 years. That regression (using returns for January 1921 through January 2001) is:

$$Return_{10} = 1.272 EY_{51} - 1.12 \quad R^2 = 0.392 \quad (3)$$

$$(6.618) \quad (-0.85)$$

Using values for January 2001 of  $EY_{11}=2.72$  and  $EY_{51}=2.05$ , the predicted one year and ten year real rates of return from equations (2) and (3) are 1.96% and 0.971% respectively.<sup>6</sup> To bring the predicted one year and ten year real rates of return up to 5 %, the S&P 500 would have to decline to 638 or 571 respectively.

We see that there is evidence that investors are myopic, looking only at current earnings in determining price, whereas it is most sensible to have a longer time period for calculating earnings for the purpose of prediction. The one-year  $R^2$ s are much smaller than the ten-year  $R^2$ s as one would expect.

Our 11-year lag is close to Shiller's postulated 10-year lag. Elsewhere, however, Shiller uses a longer lag. Shiller [1989, p.35] uses a 30-year moving average of earnings to



calculate the earnings yield that he uses to forecast one-year real returns of the S&P 500 index.<sup>7</sup>

## **V. Predicting Earnings**

To get another judgment on what lag for earnings works best in predicting future stock market conditions, we use various averages of current and past real earnings to predict earnings fifteen years in the future. Using predicted real earnings from 1890 through 2000, we find an eleven-year average gives the highest  $R^2$ . This is additional evidence that substantial time lags are appropriate in using earnings to predict the economy's future. Thus, the 11-year time lag, found in the previous section's one-year return prediction also works best for this prediction.

## **VI. Predicting Dividends**

To proceed with our goal of calculating the return from a buy and hold strategy, we need to predict real dividends for 2001 onward. Real earnings growth from 1870 through 2000 is 1.419 % per year.<sup>8</sup> From 1950 through 2000 it is 1.412% per year. From 1971 through 2000 it is 1.371% per year. This implies that the recent past is not too different from the distant past.<sup>9</sup> The corresponding figures for dividend growth are 1.016%, 0.890% and 1.194 %. Over time tax rates have increased, which may explain the slower growth rates of dividends. We postulate that tax rates will be constant in the future, so dividends in the future will grow at the same rate that earnings have grown in the past, thereby maintaining the dividend/earnings ratio.

However, the ratio of dividends to earnings measures the fraction of earnings that are not being plowed back into the firm or used to buy back stock to generate additional growth in earnings and dividends per share. If earnings and dividends are to grow at the same rates as they have done historically, dividends must be the same fraction of earnings as has been the case historically. Historically, the ratio of smoothed dividends to smoothed earnings was higher than it is today. Thus, with dividends currently so low, we would expect earnings to grow more rapidly than in the past.

Over the period January 1950 - January 2001 the ratio of dividends to an 11-year average of earnings averaged 0.5630%. In 2001 it was 0.4450 %. Only if dividends rose to  $0.5630/0.4450=1.251$  times their current level would we expect the same rate of growth of dividends and earnings per share to prevail, as was the case in the past.<sup>10</sup>

The corresponding calculation for the 51-year average (again for January 1950- January 2000) is  $0.778/0.598 = 1.302$ . For our simulations, we use the 51-year number and assume that for our simulated real rate of return calculations, real dividends used to calculate the simulation dividend yield in January 2001 rise to 1.302 times their observed level in January 2001 and then continue to grow along with earnings at the rate that prevailed for earnings historically.

Multiplying our estimate of the real dividend yield in January 2001 by 1.302 implies a value of 1.621 % for our simulation dividend yield in January 2001,  $y^s_{2001}$ . Making this adjustment lets us assume that real dividends grow at an instantaneous rate of 1.412 %

per year, thereafter. This translates into an annual growth rate (with annual compounding),  $g$ , of  $e^{0.01412}=1.422\%$ .<sup>11</sup>

## VII. Predicting the Current Return to Buying and Holding the S&P 500

Let's predict the rate of return from January 2001 onwards using a simple formula and intuition, rather than relying on the complex internal rate of return calculation that we use in most of the paper. Since we assume that the index is never sold, we can assume anything about price that we wish (so long as it does not appreciate faster than  $g + y$ ). We choose to assume that price is proportional to earnings, and that price, earnings and dividends continue to grow forever at the annual rate,  $g$ , calculated above. We assume dividends are paid throughout the year. Thus, the predicted instantaneous rate of return in January of year  $t$  is calculated as

$$r_t = g_t + y_t, \quad (4)$$

From equation (4) we acquire a predicted instantaneous real rate of return for a buy and hold strategy from January 2001 onwards of  $1.412\% + 1.621\% = 3.033\%$  per year, which translates into an annual rate of return of  $3.08\%$ /year. The determination of the hypothetical instantaneous rate of return,  $r^h$ , by the hypothetical price in 2001,  $P^h_{2001}$ , and its implied hypothetical yield  $y^h$ , is given by:

$$\begin{aligned} r^h &= G + y^h = g + y^s_{2001} * P_{2001} / P^h_{2001} \\ &= 0.01412 + .01621 * 1335.64 / P^h_{2001}. \end{aligned} \quad (5)$$

We calculate the average return from a buy and hold strategy for buying periods: January 1871-January 2001 to be 6.29 %/ year (assuming annual compounding). Jeremy Siegel [1998, p.13] calculates a historical rate of return of 7 % of the American stock market for the period 1802-1997. But Siegel's figure reflects a rise of the price earnings ratio in his final period considerably above its historical average, so as he argues in our last footnote, this 7% real return is unlikely to be repeated in the future. We transform equation (5) to yield an expression for the predicted annual real rate of return. For the ongoing rate of return to rise to 4 % or 5 % the index would have had to have fallen from its January 2001 value of 1335.64 by 35 % to 863 or 53 % to 624. For it to reach the figures of 6 % or 7%, which are closer to historical average returns, the index would have had to fall by 63% to 490 or by 70% to 404. To translate these values into current prices, one needs to raise them by the percent by which the CPI has risen since January 2001.

Shiller [2000, figure 9.1] finds that the S&P 500 index which is equal to the present value of its dividends (discounted at the historical real rate of return of the stock market at .06 % per month) is a mere 339 (based on our reading of his graph). This suggests that the appropriate value for the S&P 500 index was less than one fourth (24%) of its January 2000 level of 1425.59. To us, this seemed too huge to be imaginable. Equations (4) and (5) shed light on the intuition behind this (and our) predicted enormous declines. Each one percent fall in the index raises the dividend yield by one percent. But if the yield is low relative to the anticipated growth of real dividends, the fall has a small impact on expected real return. Only after the index has fallen substantially does the elasticity of the expected return with respect to the index become large. In Figure 3, the curve

referenced as “Favorite scenario”, and marked with diamonds tells the same story with a picture. If it were extended, it would show that the expected annual return as a function of the index becomes flat at the predicted  $g$  of 1.422%/year.<sup>12</sup>

In that figure, we also tell another story marked “Pessimistic scenario.” That calculation is identical to the Favorite calculation except that we assume that dividends continue to grow at the rate observed since 1950 of 0.890% per year.<sup>13</sup> The endnote explains why we think our “Favorite scenario” is preferable. We explain our “Optimistic scenario” calculation in Section IX.

### **VIII. The return to Buy and Hold investing in History**

Using the internal rate of return program on Microsoft Excel and assuming that the stock is purchased and never sold, we construct Figure 4. It shows the graph of the return to the buy and hold strategy versus purchase dates for Januarys from 1871 through 2001.

Drawing on the entire monthly data set, the highest figure is for a purchase in June 1932: 13.02 %. The lowest figure is for August 2000: 2.88 %. The highest monthly average real stock price in 1929 occurred in September. A purchase at that price would have yielded a 3.73 % return, considerably better than the August 2000 figure.<sup>14</sup>

### **IX. Predicting the Return to a Buy and Hold Strategy**

Now we ask how predictable is the return from a buy and hold strategy. We predict the real rate of return using the earnings yield with average earnings ranging from a one year

to a 55-year average. We find, the longer the period, the higher is the  $R^2$ . We focus on three averages: one year, because of the importance that investors attach to the one year earnings yield, and the 11 and 51 year averages which are better predictors.

The regression for the 1-year average is:

$$R = 0.387 \text{ EY1} + 3.331 \text{ Buy years 1871-2001 } R^2 = 0.368 \quad (6)$$

(8.658)    (9.184)

The regression for the 11-year average is:

$$R = 0.473 \text{ EY11} + 2.697 \text{ Buy years 1881-2001 } R^2 = 0.674 \quad (7)$$

(15.70)    (11.40)

The regression for the 51-year average is:

$$R = 0.726 \text{ EY51} + 1.914 \text{ Buy years 1921-2001 } R^2 = 0.956 \quad (8)$$

(41.58)    (16.71)

We are astounded by how well the 51-year average performs absolutely and relative to the other two. We don't have a good explanation for it, except that apparently cycles are long. Figures 5, 6 and 7 graph the data points used in equations 6, 7 and 8. They use different symbols for the observations in different time periods. Januarys in the 20<sup>th</sup> Century are denoted with the last two digits of the year. In each case, the year 2001 observation is the third point from the left.

Equation 8 and Figure 7 allow us to check the accuracy of our prediction of the real rate of return for a January 2001 decision to buy and hold. Plugging the January 2001 value of EY51 (2.05%) into equation 8 predicts a real rate of return of 3.40 % / year, 0.32 percentage points higher than our preferred estimate of 3.08 % / year.

We recognize that the real rates of return simulated for recent years depend strongly on the real rate of return postulated from 2001 on. To obtain an estimate of this real rate of return that is reasonably independent of our postulated value for it, we reestimate equation 8, using data through 1979 only. When we plug the January 2001 value of EY51 into this regression equation we predict a real rate of return for January 2001 onwards of 3.51%/year. This is 0.43% age points higher than our Favorite scenario of 3.08%. We base the curve labeled “Optimistic scenario” in Figure 3 on this calculation.

The long lag for average real earnings which maximizes  $R^2$  and the high  $R^2$  generated surprise us. It appears that there are mechanisms that only operate in the long run. Also, these calculations confirm that Jeremy Siegel’s [1998, p. 79] contention that the earnings yield explains the real rate of return has merit, but long lags increase the accuracy of the prediction.<sup>15 16</sup>

Equation 8’s regression line, shown in Figure 7, underpredicts by over half a percent per year the rates of return from buying and holding in each of the years 1947-1955. None of the other years are underpredicted by so much (although 1944 through 1946 are underpredicted by lesser amounts). We attribute this to the lower earnings during World War II and postwar years due to the reduced labor force in the private sector, reduced

private sector investment, costs of retraining labor after the war, and wartime price controls.

Equation 8's intercept is 1.918. This is substantially and significantly greater than zero, implying that even when the earnings yield is zero, the predicted real return is 1.9% per year. This flouts our initial intuition that the prediction should have been zero. But upon reflection, we realize that a stock is a claim on future as well as current dividends. Consequently, a halving of a stock price should result in less than a halving of the predicted rate of return from a stock investment, implying a positive vertical intercept.

Is the intercept larger than we should have expected? From equation 4, we would expect an intercept equal to the historical or projected growth rate of real dividends. The highest plausible figure for that is equation 5's 1.412% multiplied by 1.302 to equal 1.848%. This adjustment reflects the expectation that without any increase in the dividend payout ratio, earnings and dividends are likely to grow 1.302 times as fast as they have done in the past (the alternative simulation discussed in footnote 12).

Thus, the intercept is plausible but slightly higher than we expect. One explanation for this excess is that measurement errors in earnings or incorrect lags lower the coefficient on the earnings yield and bias up the intercept, just as the intercepts in equations 6 and 7 which use inferior lags exceed the intercept in equation 8. Alternatively, it could be that investors have additional information and bid up the price of the stock and down the



earnings yield when they have extraneous information, not based on past earnings, that future dividends are likely to be high.

## **X. Exuberance Rooted in Recency**

Examining the regression line behind Figure 1 and working with the calculations behind it indicates that at the end of 1991 real earnings were 34 % below trend. In the third quarter of 1994 they crossed it, and in December 2000 they were 45% above trend. The ten percent real earnings growth rate from January 1992 to December 2000 (calculated from a regression), if it were to continue, along with the high level of earnings relative to trend in December 2000 would more than justify the January 2001 valuation. Our analysis hinges on the idea that earnings levels and growth rates will return to their historical norms, so that the dividends they can support will not deviate markedly from those extrapolated from the historical experience. But it may be difficult for many investors to resist extrapolating from recent experience.

## **XI. Concluding Remarks**

The calculations provided here buttress Shiller's conclusion that the U.S. stock market has been experiencing an unsustainable bubble. It appears that Glassman and Hassett's [1999] prediction that the equity premium on the expected return of equities over bonds would dramatically shrink has been fulfilled. However, to us, it seems unlikely that sustained investor interest in the U.S. stock market will be maintained without an expected real return of 5 percent, (given the real interest rate on the 10-year inflation protected treasury bond exceeded 3% in November 2001). This would imply that the

valuation of the U.S. stock market must fall. Our best guess is that to obtain this yield the S&P500 needs to fall to 624 (holding the CPI constant at its January 2001 level). Our optimistic scenario calls for a smaller fall (to 786) and our pessimistic scenario calls for a larger drop (to 543).<sup>17</sup>

Our expectation of a substantial drop is buttressed by the existence of many mutual funds that charge expenses of well over 1 % of asset value. A further fall in asset values would be necessary to predict attractive rates of return for mutual fund holders. Moreover, assuming mutual fund companies hold their ratios of expenses to asset values constant, such a fall would have the additional benefit of increasing dividend yields for mutual fund shareholders.<sup>18</sup>

Our calculations serve to emphasize the importance of dividends. If there is a limit to how much the ratio of the price of the stock index to dividends can rise, then if the index is held long enough, the price at which it is sold doesn't matter for calculating the real rate of return. Moreover, there is another reason that dividends are useful. They play a role in reducing risk. If the June 1932 dividend yield of 13.8% were to prevail, investors would be compensated in a year for a 13.8% drop in the index. But, if the August 2000 dividend yield of 1.11 % were to prevail the investor would be compensated in a year for the much smaller drop of only 1.11 %.<sup>19</sup>

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## Endnotes

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<sup>1</sup> Smithers and Wright (2000) present a nice, simple argument for why the return is  $g + y$ . Suppose I have a choice between investing a dollar in an account which pays me 10%, with payments rising at 5% per year, and an alternative that pays 15% forever. I could use the alternative to replicate the cash flow from the former by pocketing 2/3rds of the interest at the end of each year and reinvesting one third. In this way, I would initially pocket 10 cents, and my assets and interest payments would be rising by 5% per year. Consequently, I would pocket the same cash flow as with the first option. Therefore, the two options must yield the same rate of return.

The reason the initial purchase price doesn't matter is the rate of return on an investment that returns money only in the terminal period, and that yields  $g + y$  for many years in the steady state and something else for only a few years, must approach  $g + y$  in the limit as the ratio of "many" to "few" becomes infinite.

<sup>2</sup> As discussed subsequently, in his chapter 9 Shiller presents a graph for the present value of real dividends and compares it to the real market price of the S&P index, an approach that is the flip side of ours. His purpose was to demonstrate that the value of the stock market based on future dividend payments varies little from year to year compared with observed price. But our approach has more utility for investors who wish to compare likely returns from stocks with past returns and the returns on bonds, especially since we remain unconvinced that there is a "right" discount rate for calculating present value.

The ultimate value of an investment doesn't matter for calculating the rate of return on the investment if the investment is held long enough. This is illustrated by our calculation that the annual internal rate of return for the S&P500 purchased in January 1871 and thrown away in January 2001 is 8.2614 % / year. If the index had been sold in 2001 the return rises only to 8.2670 % / year, less than six thousandths of a percentage point higher.

<sup>3</sup> The November 2001 figure for price is 1142. The last real earnings figure is for the year ended September 2001. The last real dividends figure is for the year ended March 2001.

<sup>4</sup> In each of the regressions that follow, we use the largest possible subset of the data. We compare  $R^2$ s for predicting the same set of observations. We then pick the length of moving average for real earnings which maximizes  $R^2$ . The longer the period in calculating the average is, the fewer observations on the dependent variable we are able to use.

<sup>5</sup> Shiller (1989, table 1.1) regresses real S&P 500 price on current real earnings and a distributed lag of earlier real earnings. He finds a positive coefficient for the former and a negative coefficient for the latter, which is consistent with our regression. It implies that high current earnings and positive earnings momentum each make investors more optimistic.

<sup>6</sup> Equal weights on real earnings are used in calculating the EYs. But, in fact, more recent earnings assume greater importance than earlier earnings, since real earnings grow through time. For example, real earnings in 2000 were 2.8 times real earnings in 1950.

<sup>7</sup> Shiller (1989, table 1.1) finds that real one-year return depends negatively on recent real earnings and positively on a distributed lag of earlier real earnings, which buttresses his conclusion that investors overreact to recent earnings and earnings momentum. This pattern of coefficients is what one would expect if investors believe that current earnings and earnings momentum predict future earnings and set price as an increasing function of predicted future earnings, when in fact a weighted average of past and current earnings predicts future earnings.

<sup>8</sup> The rates in this paragraph are instantaneous growth rates, assuming continuous compounding. To de-emphasize the role of end points we use a regression to do the calculation. We take natural logs of real earnings or real dividends, and regress this series on time. The coefficient is the instantaneous rate of growth of real dividends defined as a fraction per year.

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<sup>9</sup> Peter Lynch, Vice-Chairman of Fidelity Investments writes in an ad in the *New York Times*, (October 2, 2001, p. C3) Since World War II, “Corporate profits per share have grown over 9 % annually despite the down years. Nine percent may not sound like a lot, but consider that it means that profits mathematically double every eight years.” Lynch makes it sound as if the world is different from the ones that yielded our earnings growth figures. We don’t know where he gets his numbers. The largest figure we could get from just after World War II through June 2001, was a 7.12% average annual growth rate from June 1946. The corresponding average instantaneous growth rates between the two periods from regression analysis were 6.00 % for nominal earnings and 1.65 % for real earnings. Thus, even using this period, the real earnings growth rate is not much higher than the figure we used. In spite of Shiller’s (2000) attack on this ad, Fidelity has not changed its substance.

Glassman and Hassett (1999) also draw on Shiller’s data but use much higher growth rates for earnings and dividends than we do: 2.2% to 3% for real dividends (p.71) and 2.9% to 5.8% for real earnings (p.86). Not surprisingly, this helps them conclude that the S&P 500 index was massively undervalued in 1999.

<sup>10</sup> We estimate real dividends disbursed in each January by taking Shiller’s figure for nominal dividends paid over the most recent 12 months, including January. This we divide by the average monthly CPI over that period times 12. To incorporate the real growth of dividends over time, we multiply this figure by the factor that real dividends in January are estimated to exceed real dividends in the most recent 12 months. This is  $\exp[\text{gd} \cdot 5.5/12]$ , (gd being 0.01029, the historical rate of growth of real dividends since 1871, and 5.5 being the average number of months by which January follows the most recent 12 months). We use the same procedure for calculating dividends disbursed in each month. These figures we need for calculating real rates of return for both the buy-and-hold strategy and the reinvestment strategy.

<sup>11</sup> This is the same correction that Shiller (2000) makes in his footnote 24 to his chapter 9. Shiller’s (2000) Figure 9.1 compares the stock price and dividend present value from 1871 through the beginning of 2000. In calculating present values Shiller uses a discount rate of 0.6 % per month. Thus, we use the same approach Shiller does, except he calculates the stock price that yields the required rate of return, while we calculate the rate of return that equates the present value of dividends to the price of the index.

<sup>12</sup> In our simulations, we assumed that the dividend jumps in January 2001, with the growth rate held at its historical level. An alternative simulation would have assumed that the dividend stays constant, but the growth rate jumps, as the excess dividends above their historical levels are used to buy back stock. Assuming further that the ratio of the stock price to dividends and earnings stays the same after January 2001 as it was in the estimated equilibrium of January 2001 makes the terms of the buyback predictable. It is not too hard to show that it also makes the predicted rate of return in the alternative simulation identical to what we calculated in the text.

<sup>13</sup> Shiller (2000, footnote 24 of chapter 9) calculates the future growth rate in this way, except he uses the average growth rate of real dividends all the way back to 1871. We prefer to predict using the growth rate of real earnings because of the following argument.

Suppose that the S&P500 consists of equal weightings of two stocks: Growth and Value, Growth pays no dividends but experiences real earnings growth at 5% per year. Value experiences no growth in real dividends or earnings, but pays real dividends of 5% per year. Assuming price/earnings ratios do not change, both stocks generate the same real rate of return. Consequently, the S&P500 also returns 5%. This could be written as the dividend yield of the index, which is 2.5% plus the rate of growth of real earnings which is 2.5%, for a total of 5%. If instead we substituted the rate of growth of real dividends we would calculate a rate of return on the index of only 2.5%.

<sup>14</sup> Our assumption about dividends after 2000 does not affect the internal rate of return calculations for early investments much. For example, the annual internal rate of return for purchasing the S&P500 in January 1871 and selling it in January 2001 is 8.2670 % / year. Under our assumption that it is held forever the rate falls to 8.2627 %.

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<sup>15</sup> Tom Houghton suggests that changes in accounting rules limit the comparability of earnings in one period to that in another. In particular he suggests that many items that would have been considered extraordinary in the past now have to be charged against earnings, thereby raising the PE ratio above what it has been historically. Consequently, he argues, the stock market may not be as overvalued as it appears. He also suggests that the mix of the S&P 500 has changed over time with industrial stocks less important today than they once were. To the extent this has increased of high PE-type stocks relative to low PE –type stocks, the S&P500 may seem to be increasingly overvalued when it is not. Moreover, as Siegel (1998) points out, changes in the inflation rate affect the quality of earnings.

Others have argued that the sale of stock options to employees, which has increased in importance in recent years, constitutes a cost to the firm that are not immediately charged to earnings. The same is true of some pension liabilities. These would cut in the other direction.

<sup>16</sup> Hall (2001) argues that changes in the discount rate and actual future growth rates of cash flow explain a substantial proportion of changes in stock prices, which constitutes evidence against the view that stock prices are irrational. This does not conflict with the evidence presented here that the earnings yield calculated with a long trailing average of real earnings does a good job of predicting the real rate of return from buying and holding the S&P 500 index.

<sup>17</sup> In deriving Figure 3, we made assumptions that need highlighting. In each scenario, we took the growth rate of dividends to be given and raised the simulation dividend yield above the estimated one. An alternative would have been to perform the simulation, equating the simulation dividend yield to the estimated one and increasing the growth rate, to leave the simulated real return from a buy and hold strategy at the initial price of the index unchanged. However, the latter simulation would have made the return less sensitive to the price of the index. Thus, we have deliberately chosen the high sensitivity alternative, which makes the “Favorite” curve in Figure 3 steeper than it would be otherwise. This shrinks the magnitude of the fall in the S&P500 needed to raise the hypothetical yield to a given level. Thus, we have minimized the dramatic impact of our simulations.

<sup>18</sup> Siegel (1999, p.13) argues that the advent of mutual funds and the reduction in the cost of investing in them, especially the availability of low cost index funds, has substantially reduced the cost of maintaining a diversified portfolio. This, he argues, should make investors content with lower real rates of return from equity indexes than were observed historically. He writes: “Given transaction costs and inadequate diversification, I assume that equity investors experienced real returns more in the neighborhood of 5 % to 6 % over most of the nineteenth and twentieth century rather than the 7 % calculated from indexes. He notes (p.14): “If future dividends grow no faster than they have in the past, forward-looking real stock returns will be lower than the 7 % historical average.” He then argues that the increased appeal of equity investment may be responsible for the bull market, which has temporarily increased equity returns, but that these high returns are not likely to be repeated in the future. Campbell (2001, p.8) provides a rough guess for the long term geometric real return on equities, after the current bubble has burst of 5.0% - 5.5%. Diamond (2001, p.22) discusses how the reduced cost of investing in mutual funds, the spread of stock ownership, changed tax laws and increased investors’ understanding of stock markets should reduce the equity premium. Shoven (2001) picks a figure of 6% to 6.5% for the long run real rate of return on the stock market, arguing that real dividends will grow as fast as real GNP. Given that new firms are continually born, we expect, contrary to Shoven, that the dividend growth of existing firms will be less than the growth rate of real GNP.

<sup>19</sup> For a given initial real expected real rate of return, an adverse shock to profitability should result in a greater proportional fall in the stock price when the initial dividend yield is high. Thus, high dividends do not shrink risk. It is just that a given variability of stock prices imposes more risk, the less the dividend yield is.