EC 262P: Population Economics
Fall 2002

Static Economic Models of Fertility

1. Simple Demand for Children Model

(1) $$U = U(n, s)$$

where n is the number of children and s is the composite consumption good.

Parents choose n and s so as to maximize (1) subject to

(2) $$I = \pi_s s + p_n n$$

where I is the household's income,

p_n per unit "price of children,

π_s per unit price of the composite commodity.

Demand-for-Children Function:

(3) $$n = N(p_n, I)$$

2. The Quality-Quantity Model

2.1 Becker's 1960 Model

How to account for the negative relationship between fertility and income in both time series and cross section?

Answer: Parents care about both quantity and quality of children.

Parental preferences:

(4) $$U = U(n, q, s)$$

where

n number of children,

s parents' standard of living,

q "quality" per child.

Budget constraint:

(5) $$I = \pi_c nq + \pi_s s,$$

where

π_c price index of goods and services devoted to children

Budget constraint is nonlinear due to q and n.

Income elasticities of demand for n, q and s must satisfy:

(6) $$\alpha(\varepsilon_n + \varepsilon_q) + (1-\alpha)\varepsilon_s = 1$$

where

α is the share of family income devoted to children

ε's denote income elasticities.

If children are normal goods, i.e., total expenditures on children increase with income, then $\varepsilon_n + \varepsilon_q > 0$)

But, possible that $\varepsilon_n < 0$, if ε_q large enough.

2.2 Willis (1973) and Becker-Lewis (1973) Quantity-Quality Models of Fertility

Implications of nonlinearity in (5) explored.

Maximizing (4) subject to (5) yields

(7) $$MU_n = \lambda q \pi_c = \lambda p_n; \; MU_q = \lambda n \pi_c = \lambda p_q$$

where

p's are marginal costs or shadow prices of n and q

λ is marginal utility of income.

Note:

p_n is increasing function of q

and p_q is increasing function of n.

Shadow prices are endogenous!

See Figure 7.

Figure 7 Interaction of the Demand for Quality and Quantity of Children

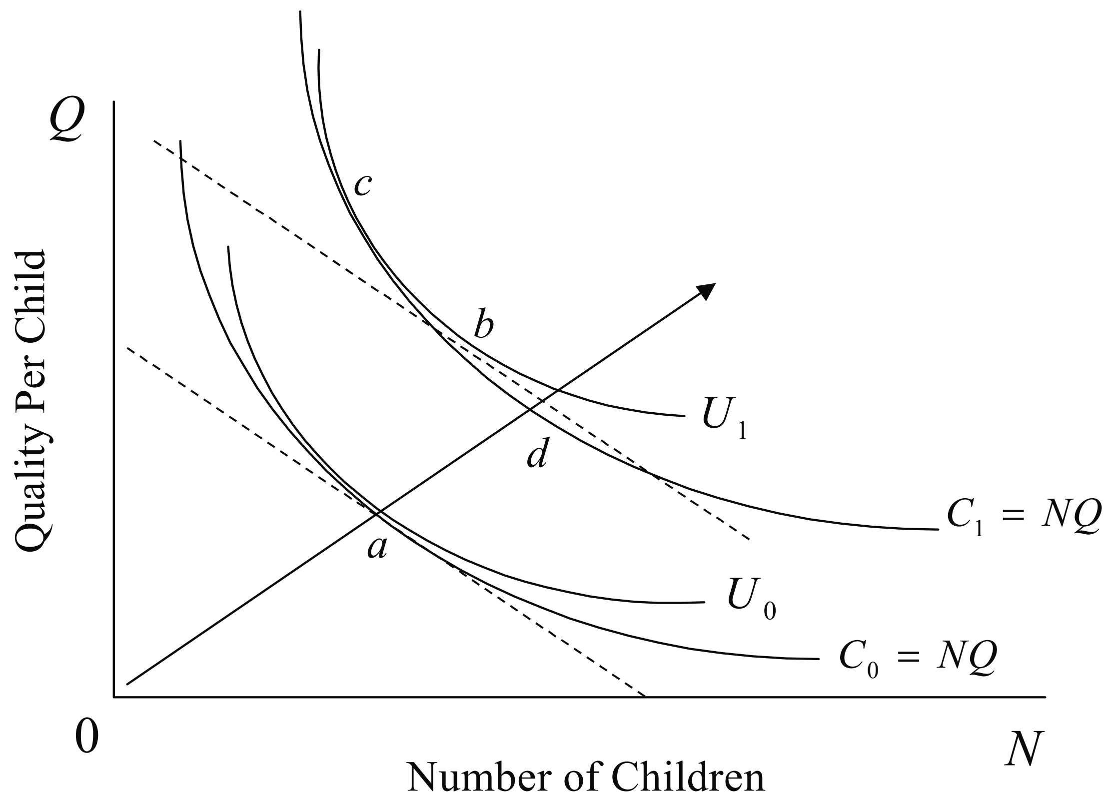


Figure 7 Discussion:

At equilibrium, U_0 is tangent to the budget constraint,

$C_0 = nq = (I - \pi_s s(\pi_c,\pi_s,I))/\pi_c$,

where c_0 is household's real expenditure on children

$s(\pi_c,\pi_s,I)$ is demand for parents' standard of living.

(Note nonlinearity of this budget constraint.)

Consider what happens when income increases:

Case where $\varepsilon_q = \varepsilon_n$.

Case where $\varepsilon_q > \varepsilon_n$ (quality is more income elastic than quantity).

Generalization:

Consider existence of costs of n that are not dependent on q and visa versa. Generalized budget constraint

(8) $$I = \pi_n n + \pi_q q + \pi_c nq + \pi_s s$$

where π_n and π_q are these independent costs and

$$p_n = \pi_n + \pi_c q$$

$$p_q = \pi_q + \pi_c n.$$

and think of π_n as opportunity cost of fertility control.

Consider exogenous introduction of new contraceptive methods that *reduces* cost of averting births and *increases* π_n (because no longer lose sexual pleasure with sexual intercourse).

What happens when p_n is increased to quantity and quality demanded?

Alternatively, consider decrease in π_q due to increase in parental education (i.e., more parental education improves efficiency of producing better children).

2.3 Time Allocation and the Demand for Children

Second major reason for a negative relationship between income and fertility:

Higher income is associated with a higher cost of female time, either because of increased female wage rates or because higher household income raises the value of female time in nonmarket activities.

Simple Model of Women's Labor Supply and Fertility by Willis (1973):

Consider home production functions for adult standard of living, s, and children, n and q.

Simplifying assumptions:

1. Only wife participates in the production of household commodities while husband fully specialized market work and his income, H, is treated as exogenous. Total family income is

$$I = H + wL$$

   where w is the wife's real wage and L is her labor supply.

2. Utility depends on adult consumption and "child services,"

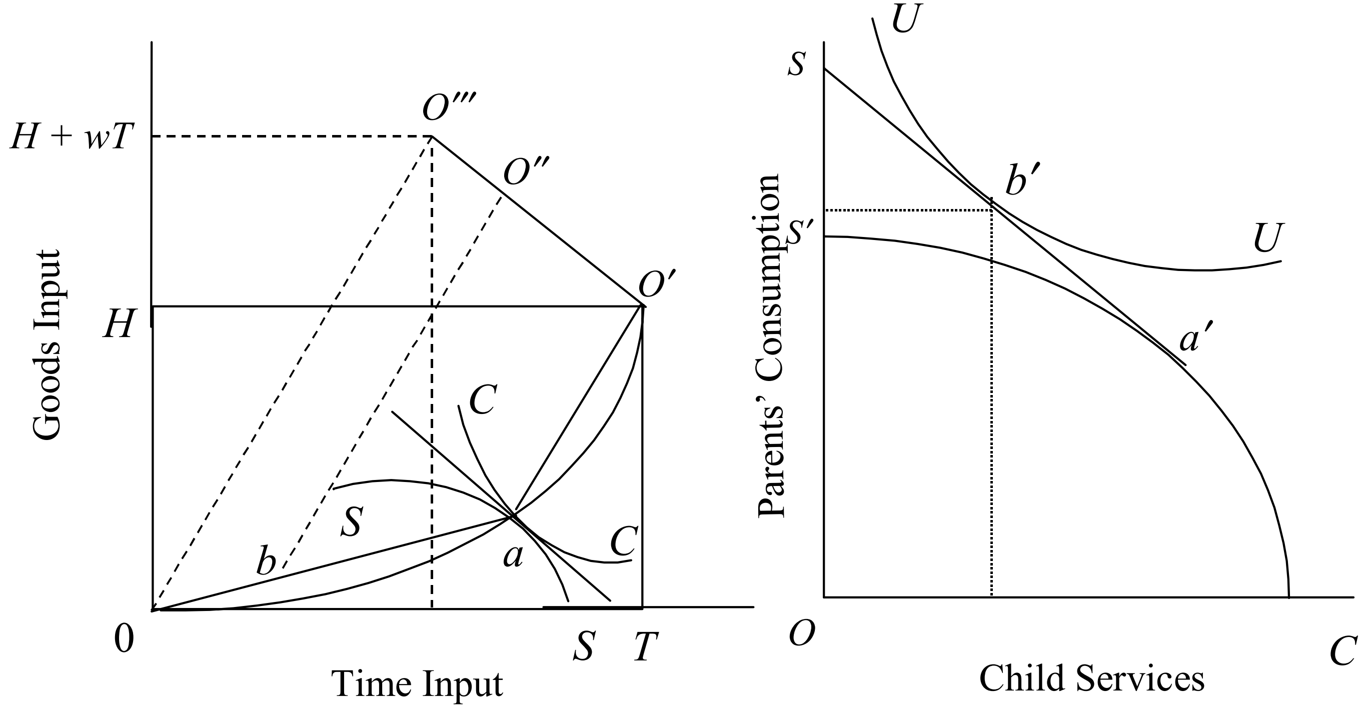
$$s = g(t_s, x_s)$$

$$c = nq = f(t_c, x_c)$$

   where production technology for children is time-intensive (in woman's time) relative to the technology for parents' standard of living.

See Figure 8.

Figure 8: Time Allocation and Fertility Decisions



A. Input Space

B. Output Space

At Point a, $\hat{w}$, shadow price of the wife's time, $\hat{w}$, is equal to:

$$\hat{w} = f_t/f_x = g_t/g_x.$$

Corresponding outputs of c and s at point a' on the production possibility frontier in Panel B of Figure 8.

Because children are relatively time intensive, an increase in the price of the time input leads to an increase in the relative cost of the time intensive output.

Relative shadow price of children,

$$\pi_c/\pi_s,$$

(slope of the production possibility frontier in Panel B) tends to increase as the output children rises above the level indicated at point a'.

Consider what happens if woman enters market and receives wage w and generates income for the household:

Move to point b in Panel A and to point b' in Panel C of Figure 8, where b' is the household's optimal choice. Note that relevant Edgeworth Box is now OO'', devoting some woman's time to market.

Predicted Effects of exogenous changes in woman's wage (w) and husband's income (H).

See Figures 9 and 10.

Figure 9: Effect of an Increase in the Female Wage

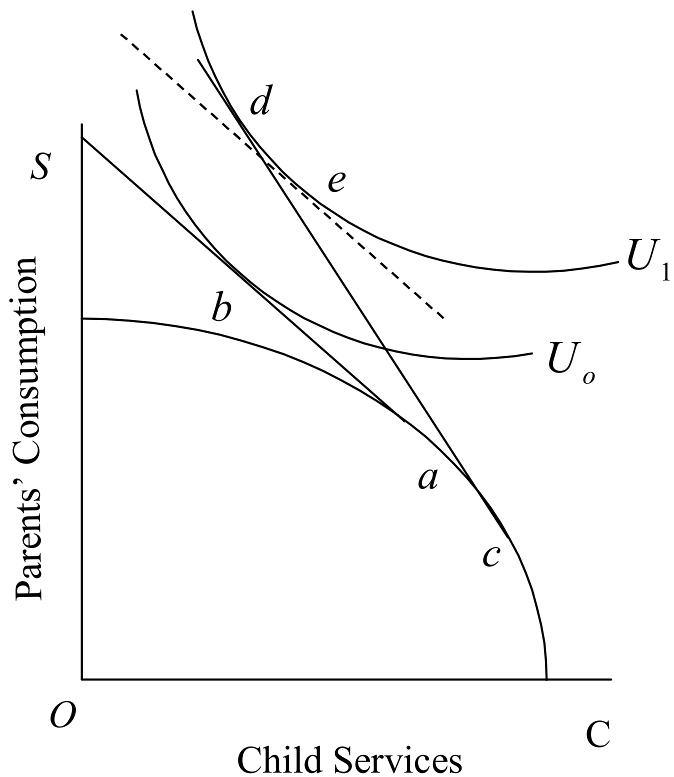


Figure 10: Effect of an Increase in the Husband's Income