

Life Cycle Labor Supply Models

1. Motivation

Recall simple static labor supply model of hours of work

$$H_{it} = \alpha + \beta w_{it} + \delta Y_{it}^N + \varepsilon_{it} \quad (1)$$

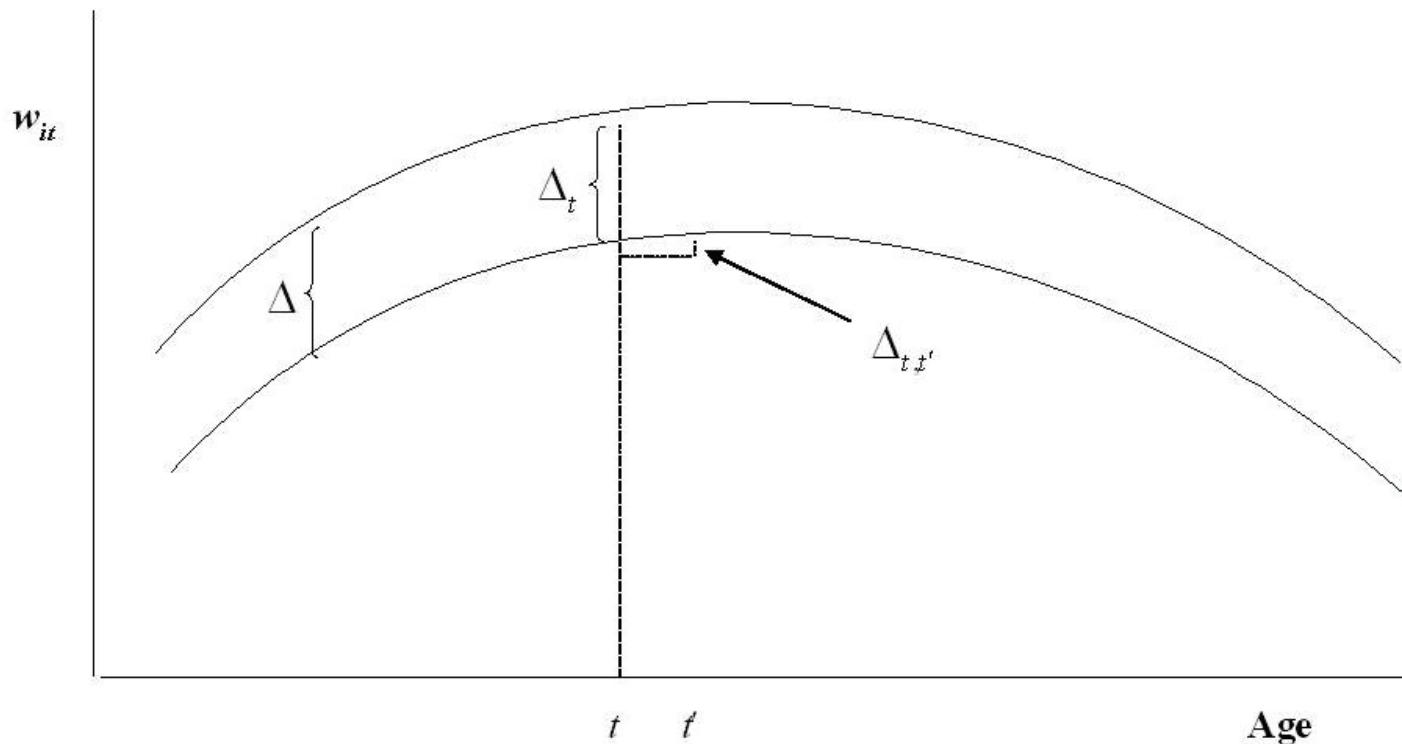
where Y_{it}^N is non-labor income, including savings or returns from assets.

- But is Y_{it}^N exogenous?
 - Focus of important literature in economics has been on the intertemporal allocation of consumption and savings:
 - **Income Smoothing Motives or Permanent Income Hypothesis:** Desire of consumers to smooth consumption over life cycle lead to motives for smoothing income over life cycle in order to support such consumption plans.
 - **Other Motives for Savings, e.g., precautionary savings:** Saving to provide insurance for being able to support consumption in the future in face of uncertainty about income sources over time.

Both motives suggest that Y_{it}^N is endogenously determined over the life cycle.

- Which wage elasticity do we want to estimate? Or, the effect of what type of wage change do we want to estimate?
 - β measures change in hours with respect to wages.
 - But different type of wage changes. See *Alternative Sources of Wage Variation*.
 - Different sources of wage change imply different wage elasticities for Labor supply.
 - Long run elasticities: Effects of permanent changes of wages on hours.
 - Short-run elasticities: Effects of temporary (or possibly evolutionary changes in wages over life cycle) produce “intertemporal substitution of hours of work (Lucas & Rapping, 1970)

Alternative Sources of Wage Variation



Δ is *permanent* wage change, shift in entire wage profile

Δ_t is *transitory* wage change, shift in wages just at time period t

$\Delta_{t,t'}$ is an *evolutionary* wage change, i.e., movement along a wage profile

2. Simple Life Cycle Labor Supply under Perfect Certainty (MacCurdy, 1981, Heckman and MacCurdy, 1980)

Maximize:

$$U \equiv \sum_{t=0}^N (1 + \rho)^{-t} U(C_t, L_t; x_t, \varepsilon_t) \quad (2)$$

or, in special case of $U(C_t, H_t; x_t, \varepsilon_t)$ is additively separable:

$$U \equiv \sum_{t=0}^N (1 + \rho)^{-t} [G(C_t; x_{1t}, \varepsilon_{2t}) + J(L_t; x_{1t}, \varepsilon_{2t})] \quad (2')$$

subject to:

$$A_0 + \sum_{t=1}^N (1 + r)^{-t} w_t H_t - \sum_{t=1}^N (1 + r)^{-t} p_t C_t \quad (3)$$

or

$$A_{t+1} = (1 + r) [A_t + w_t H_t - p_t C_t] \quad (3')$$

where

ρ is the consumer's rate of time preference

r is the interest or discount rate.

$$L_t + H_t = 1.$$

First Order Conditions:

$$\frac{\partial U}{\partial C_t} = U_C = \left(\frac{1+\rho}{1+r} \right)^t \lambda_0 p_t = \lambda_t p_t \quad (4)$$

$$\frac{\partial U}{\partial L_t} = U_L \geq \left(\frac{1+\rho}{1+r} \right)^t \lambda_0 w_t = \lambda_t w_t. \quad (5)$$

where λ_t is the *marginal utility of wealth* as of period t .

Note that intertemporal condition also holds:

$$\frac{\partial U / \partial C_{t+1}}{\partial U / \partial C_t} = \frac{\lambda_{t+1}}{\lambda_t} = \left(\frac{1+\rho}{1+r} \right) \frac{p_{t+1}}{p_t} = \left(\frac{1+\rho}{1+r} \right) \quad (6)$$

where the latter equality holds if $p_t = p = 1$, for all t . For convenience maintain this assumption in what follows.

Note that (6) implies that consumption increases (decreased) from t to $t+1$ as $(1+\rho) < (>) (1+r)$.

Furthermore, (6) implies that

$$\lambda_{t+1} = \left(\frac{1+\rho}{1+r} \right) \lambda_t \quad (7)$$

which is the Euler equation that determines intertemporal consumption and, thus, savings in period t .

Two-Stage Budgeting Approach to Solution of Model:

Stage 1: Allocate wealth across time periods so as to maximize lifetime utility.

Stage 2: Within each period, allocate C_t and H_t so as to maximize within-period utility subject to wealth allocation.

Doing *Stage 2* first, one allocates C_t and H_t subject to some full income, M_t , which implies indirect utility function $V_t(M_t, w_t)$. Then inserting V_t 's into U and solve for M_t 's that maximizes U .

This solution strategy implies that per-period (Marshallian) demand functions look like per-period static demand functions, except they depend on M_t , where

$$\begin{aligned} M_t &= rA_{t-1}^* + \Delta A_t^* + w_t T \\ &= Y_t^N + \Delta A_t^* + w_t \cdot 1 \\ &= p_t C_t + w_t L_t \end{aligned} \tag{8}$$

where Y_t^N is what we referred to as non-labor income, or the period t income stream from the asset A_{t-1}^* and $1 = L_t + H_t$.

Thus, *Stage 2* optimization problem can be written as:

$$\max_{C_t} U(C_t, L_t; x_t, \varepsilon_t)$$

subject to:

$$p_t C_t = rA_{t-1}^* + \Delta A_t^* + w_t H_t$$

Note that the difference between static and dynamic specification of M_t is ΔA_t^* , the adjustment in assets over t . Furthermore, it follows that ΔA_t^* is endogenously determined by the *Stage 1* intertemporal optimization problem.

Frisch Demand Functions Solution Strategy:

Assuming interior solutions, solve (4) and (5) for C_t and H_t , as a function of p_t and w_t and λ_t to get *Frisch*—or *marginal-utility-of-wealth-constant*—demand functions for consumption and leisure (hours of work):

$$\begin{aligned} C_t &= C(\lambda_t, \lambda_t w_t; x_t, \varepsilon_t) \\ &= C(\lambda_0 \theta^t, \lambda_0 \theta^t w_t; x_t, \varepsilon_t) \\ L_t &= L(\lambda_t, \lambda_t w_t; x_t, \varepsilon_t) \end{aligned} \tag{9}$$

$$\begin{aligned} &= L(\lambda_0 \theta^t, \lambda_0 \theta^t w_t; x_t, \varepsilon_t) \\ &= 1 - H_t \end{aligned} \tag{10}$$

or, when U is contemporaneously separable in C_t and H_t ,

$$\begin{aligned} C_t &= C(\lambda_t; x_t, \varepsilon_t) \\ L_t &= L(\lambda_t w_t; x_t, \varepsilon_t) \end{aligned} \tag{9'}$$

$$\text{where } \theta^t = \left(\frac{1+\rho}{1+r} \right)^t.$$

Given concavity of preferences in its arguments, it follows that:

1. Condition (4) implies that λ_0 (and λ_t) is endogenous.

2. $\frac{\partial \lambda_0}{\partial A_0} \leq 0$, i.e., declining MU of wealth.
3. $\frac{\partial \lambda_0}{\partial w_t} \leq 0$, i.e., changes in w_t generate wealth effects.
4. $\left. \frac{\partial L_t}{\partial w_t} \right|_{\lambda_0 = \text{constant}} \leq 0$, effect of a movement along wage profile.
5. $\frac{\partial L_t}{\partial \lambda_0} \leq 0$, effect of change in marginal utility of wealth.

- Recall alternative sources of wage variation. Coupled with above model, it follows that:
 - *Permanent wage changes* generate income and substitution effects, so sign is ambiguous.
 - *Transitory wage changes* generate substitution and income effects, but note that income effect is distributed over entire life cycle in this model, through its impact on λ_0 .
 - *Evolutionary wage changes* generate just substitution effects, as they hold λ_0 constant.

The above follows from the fact that

$$\lambda_0 = \lambda(w_1, w_2, \dots, w_T; x_1, \dots, x_T, \varepsilon_1, \dots, \varepsilon_T) \quad (11)$$

2.1 *Labor Force Participation Decision*

From (5) it follows that:

$$\frac{\partial U/\partial L_t|_{L=1}}{\lambda_0} = \frac{J'(1)}{\lambda_0} \equiv w_t^* > \theta^t w_t \quad (12)$$

if don't work in period t .

- Consider cases of effects of different types of wage changes.
 - Note how life cycle setting changes the LFP decisions. Now shadow price no longer “separates” from wage changes, since some wage changes change λ_0 .
 - See Heckman and MaCurdy (1980) for discussion of effects of wage changes on lifetime labor supply and labor force participation. In general, these two objects are different and, more importantly, respond differently to various types of changes in wages.
 - Moreover, current hours of work and LFP decisions depend on wage changes that occur in other periods.

2.2 Estimation of Life Cycle Labor Supply and LFP Models from Frisch Demand Functions

Let

$$U \equiv \sum_{t=0}^N (1 + \rho)^{-t} \left[\gamma_{1t} C_t^\phi + \gamma_{2t} L_t^\alpha \right] \quad (13)$$

It follows that

$$L_t = \begin{cases} \left[\frac{1}{\gamma_{2t}\alpha} \theta^t \lambda_0 w_t \right]^{1/(\alpha-1)} & \text{, if person works,} \\ \bar{L}, & \text{otherwise} \end{cases} \quad (14)$$

or

$$\ln(\bar{L} - H_t) = \begin{cases} \frac{1}{\alpha-1} [\ln \lambda_0 - \ln \alpha + (\rho - r)t - \gamma_{2t} + \ln w_t], & \text{if person works,} \\ \ln \bar{L}, & \text{otherwise} \end{cases} \quad (15)$$

where assume that $\ln \left(\frac{1+\rho}{1+r} \right) \approx \rho - r$.

Let the life cycle wage process be given by:

$$\ln w_t = x_t \beta_2 + \varepsilon_{2t} \quad (16)$$

and

$$\begin{aligned}\gamma_{2t} &= z_t \beta_1 + \varepsilon_{1t} \\ \varepsilon_{1t} &= \eta_1 + u_{1t} \\ \varepsilon_{2t} &= \eta_2 + u_{2t}\end{aligned}\tag{17}$$

where u_{jt} is a random variable with mean zero and variance σ_{jj} and $E(u_{1t}, u_{2t}) = \sigma_{12}$ if $t = t'$ and = 0 otherwise.

Let $\delta \equiv 1/(\alpha-1)$. What is δ ?

Recall that λ_0 is endogenous, that is, it is correlated with ε 's. So, can't treat it as random variable that is uncorrelated with these disturbance terms. But, can treat it as a *fixed effect*. Notice that this model motivates why one *has to use a fixed effect*.

In the case where LFP is always = 1, it follows from (15) that

$$\Delta \ln H_t = (\rho - r) + \delta \Delta \ln w_t + \Delta z_t \beta_1 + \Delta \varepsilon_{1t} \quad (18)$$

this is MaCurdy's model for men. It can be estimated using OLS if one assumes $\Delta \ln w_t$ is exogenous or, one is concerned about the endogeneity of $\Delta \ln w_t$, by IV.

In the case where LFP not always = 1, as is true for women, Heckman and MaCurdy formulate maximum likelihood estimators, based on:

$$\ln(\bar{L} - H_t) = \begin{cases} f + \delta(\rho - r)t - z_t \beta_1 \delta + x_t \beta_2 \delta + V_{1t}, & \text{if } V_{1t} \leq -f - \delta(\rho - r)t - z_t \beta_1 \delta + x_t \beta_2 \delta + \ln \bar{L} \\ \ln \bar{L}, & \text{if } V_{1t} > -f - \delta(\rho - r)t - z_t \beta_1 \delta + x_t \beta_2 \delta + \ln \bar{L} \end{cases} \quad (19)$$

and

$$\ln w_t = \eta_2 + z_t \beta_2 + V_{2t}, \text{ if } V_{1t} \leq -f - \delta(\rho - r)t - z_t \beta_1 \delta + x_t \beta_2 \delta + \ln \bar{L} \quad (20)$$

where

$$f = \delta(\ln \lambda_0 - \ln \alpha - \eta_1 + \eta_2)$$

and

$$V_{1t} = \delta(u_{2t} - u_{1t})$$

$$V_{2t} = u_{2t}$$

where note that f and η_2 are treated as *fixed effects*.

What about estimating other wage effects?

- Write out expression for $\ln\lambda_t$ and substitute for life cycle wages, $\ln w_{it}$, and non-labor income.

TABLE I
Mean values of variables for the full sample

	1968	1969	1970	1971	1972	1973	1974	1975
Family income exclusive of wife's earnings (deflated by consumer price index)	$1\cdot14 \times 10^4$	$1\cdot22 \times 10^4$	$1\cdot3 \times 10^4$	$1\cdot4 \times 10^4$	$1\cdot45 \times 10^4$	$1\cdot6 \times 10^4$	$1\cdot68 \times 10^4$	$1\cdot76 \times 10^4$
Total number of children	2.82	2.85	2.86	2.88	2.88	2.90	2.90	2.90
Number of children less than 6	0.27	0.22	0.18	0.14	0.12	0.09	0.06	0.05
Children living in family unit	1.63	1.53	1.44	1.33	1.21	1.08	0.98	0.90
Expected number of children	0.09	0.07	0.05	0.04	0.03	0.02	0.01	0.02
Childbearing finished	0.94	0.95	0.96	0.97	0.98	0.99	0.99	1.00
Number of years to last birth ^a	0.28	0.21	0.17	0.13	0.11	0.09	0.08	0.07
Did wife work before marriage?	0.72	0.72	0.72	0.72	0.72	0.72	0.72	0.72
Head retired or disabled? (1 = yes, 0 otherwise)	0.084	0.098	0.098	0.12	0.14	0.17	0.20	0.23
Future work expectations of ^b wife	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
Wife's education (years)	11.7	11.7	11.7	11.7	11.7	11.7	11.7	11.7
Does the wife have an advanced degree?	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
Hourly wage of working wives (in \$) (deflated by consumer price index)	2.37	2.43	2.47	2.60	2.57	2.61	2.68	2.54
Head's annual hours of unemployment ^c	25.7	21.0	38.1	33.3	20.1	21.6	26.2	31.8
Labour force participation rate	0.49	0.50	0.47	0.45	0.44	0.45	0.47	0.47
Age of wife	44.9	45.9	46.9	47.9	48.9	49.9	50.9	51.9
Number of observations	672							

NOTES:

- (a) This is the number of years to the last birth recorded in the sample. This variable should be accurate as a measure of years left to the last birth in the life cycle in view of the fact that the *youngest* woman in the sample is 38 years old in the last year of the sample. The mean of this variable declines over time as women near the end of their childbearing period.
- (b) A dummy variable = 1 if the wife plans to work in the future ("future" is unspecified as to its occurrence).
- (c) Unemployment hours are given only if the head is not disabled or retired.
- (d) Participation is defined by a dummy variable that equals one if a woman works for money in the year.

TABLE II
Mean values of variables for the effective sample

	1968	1969	1970	1971	1972	1973	1974	1975
Family income exclusive of wife's earnings (deflated by consumer price index)	1.06×10^4	1.09×10^4	1.3×10^4	1.3×10^4	1.34×10^4	1.1×10^4	1.1×10^4	1.3×10^4
Total number of children	2.80	2.82	2.84	2.85	2.86	2.87	2.87	2.87
Number of children less than 6	0.28	0.23	0.18	0.14	0.11	0.08	0.06	0.04
Children living in family unit	1.80	1.66	1.55	1.43	1.30	1.16	1.05	0.96
Expected number of children	0.09	0.07	0.05	0.04	0.03	0.02	0.02	0.02
Childbearing finished	0.94	0.96	0.97	0.98	0.985	0.99	0.99	0.99
Birth in current year	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01
Number of years to last birth ^a	0.29	0.23	0.19	0.17	0.14	0.13	0.11	0.11
Did wife work before marriage?	0.84	0.84	0.84	0.84	0.84	0.84	0.84	0.84
Head retired or disabled? (1 = yes, 0 otherwise)	0.06	0.07	0.07	0.09	0.10	0.13	0.15	0.18
Future work expectations of ^b wife	0.64	0.64	0.64	0.64	0.64	0.64	0.64	0.64
Wife's education (years)	12.1	12.1	12.1	12.1	12.1	12.1	12.1	12.1
Does the wife have an advanced degree?	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
Hourly wage of working wives (in \$) (deflated by consumer price index)	2.37	2.43	2.47	2.60	2.57	2.61	2.68	2.54
Head's annual hours of unemployment ^c	21.4	18.8	42.3	30.5	18.6	24.0	30.4	37.1
Hours worked in the year by working wives	1.3×10^3	1.35×10^3	1.36×10^3	1.37×10^3	1.35×10^3	1.37×10^3	1.34×10^3	1.33×10^3
Labour force participation rate	0.72	0.74	0.71	0.67	0.69	0.67	0.69	0.70
Age of wife	43.2	44.2	45.2	46.2	47.2	48.2	49.2	50.2
Number of observations	452							

NOTES:

See Table I for definitions of variables.

TABLE III
Maximum likelihood estimates of the model
(asymptotic normal statistics in parentheses)

Parameters	Unconditioned system $\bar{L} = 8760$	Conditioned system $\bar{L} = 8760$
ϕ Parameters (Effects on household production and preferences)		
Number of children less than age 6	$2.2 \times 10^{-2}(4.3)$	$2.2 \times 10^{-2}(5.8)$
Family income, exclusive of wife's earning	$5.1 \times 10^{-9}(0.01)$	$5.06 \times 10^{-9}(0.03)$
Number of children in the family unit	$1.4 \times 10^{-2}(8.9)$	$1.4 \times 10^{-2}(17.1)$
Wife's age ($r - \rho$)	$1.35 \times 10^{-2}(18.4)$	$1.4 \times 10^{-2}(28.0)$
Head's hours unemployed	$1.4 \times 10^{-8}(0.00)$	$1.4 \times 10^{-8}(0.03)$
Head retired or disabled?	$2.8 \times 10^{-2}(4.3)$	$2.7 \times 10^{-2}(10.5)$
β Parameters (Effects on ln wage rates)		
Local labour market unemployment	$-5.83 \times 10^{-5}(-0.11)$	$-5.8 \times 10^{-5}(-0.14)$
Experience (age-schooling - 6)	$3.42 \times 10^{-2}(50.6)$	$3.44 \times 10^{-2}(64.6)$
Experience squared	$-4.05 \times 10^{-4}(-22.5)$	$-4.0 \times 10^{-4}(-31.2)$
Interequation correlation ($\omega_{12}/(\omega_{11}\omega_{22})^{1/2}$)	$-0.29(-16.6)$	$-0.29(-16.65)$
Standard deviation in reduced form hours of work	$0.28(93.9)$	$0.28(130.5)$
Standard deviation in wage rate	$0.59(143.1)$	$0.59(136)$
α (substitution parameter)	$1.4 \times 10^{-3}(49)$	$1.4 \times 10^{-3}(68.5)$
Fixed effects		
Hours (ln $\lambda(0) - \eta_1$)		
Mean	-20.5	-20.5
Standard deviation	0.52	0.56
Wage (η_2)		
Mean	4.75	4.75
Standard deviation	0.52	0.53
Log likelihood value	5.95×10^3	6.01×10^3

TABLE IV

Regressions of estimated fixed effects on life cycle variables and variables that remain fixed over the sample period
 (Fixed effects generated from the model with parameter estimates reported in Table III. "t" statistics in parentheses)

Variables	Unconditioned system $\bar{L} = 8760$	Conditioned system $\bar{L} = 8760$
<i>Dependent variable: structural hours of work fixed effect ($\ln \lambda(0) - \eta_1$)</i>		
Intercept	-20·4	-20·5
Wife's education (years)	-0·055(-4·8)	-0·057(-4·8)
Wife has advanced degree (dummy = 1 if yes)	-0·324(-2·7)	-0·38(-2·9)
Family income (8 yr. avg.)	$-1·08 \times 10^{-5}(-3·1)$	$-1·2 \times 10^{-5}(-3·4)$
Expected number of children (8 year average)	0·625(2·05)	0·64(2·1)
Children less than 6 (8 year average)	0·11(1·43)	0·07(0·86)
Childbearing finished (8 year average)	0·55(1·16)	0·52(1·1)
Number of years to last birth (8 year average)	-0·048(-1·3)	-0·054(-1·4)
Wife worked before marriage (8 year average)	-0·051(-0·87)	-0·058(-0·99)
Numbers of children (8 year average)	0·033(2·5)	0·032(2·33)
Mean experience (age-schooling - 6)	0·012(4·2)	0·013(3·8)
Mean unemployment hours	$8·4 \times 10^{-5}(0·3)$	$8·45 \times 10^{-5}(0·3)$
Mean retired/disabled	-0·66(-1·1)	-0·54(-0·84)
R^2	0·31	0·325
<i>Dependent variable: ln wage fixed effect (η_2)</i>		
Intercept	3·8	3·77
Wife's education (years)	0·07(6·75)	0·075(6·9)
Wife worked before marriage	0·056(0·91)	0·062(1·03)
County unemployment rate (8 year average)	0·013(0·96)	0·02(1·5)
Expectations of future work (8 year average)	0·08(1·7)	0·047(0·90)
Wife has advanced degree (dummy = 1 if yes)	0·3(2·3)	0·36(2·7)
Mean experience (age-schooling - 6)	$-0·4 \times 10^{-3}(-0·294)$	$-3·4 \times 10^{-3}(-0·24)$
Mean experience squared	$-1·3 \times 10^{-5}(-0·05)$	$-4·4 \times 10^{-5}(-0·19)$
R^2	0·23	0·251

3. Life Cycle Labor Supply under Uncertainty (Altonji, 1986 and Blundell and MaCurdy, 1999)

3.1 Model Specification

Now future wages, prices and interest rates are uncertain. (Could be that there are preference shocks as well.)

Let p_t , w_t , and r_t be realized at beginning of period t .

F.O.C.

$$\frac{\partial U}{\partial C_t} = U_C = \left(\frac{1+\rho}{1+r} \right)^t \lambda_0 p_t = \lambda_t p_t \quad (4)$$

$$\frac{\partial U}{\partial L_t} = U_L \geq \left(\frac{1+\rho}{1+r} \right)^t \lambda_0 w_t = \lambda_t w_t. \quad (5)$$

$$\lambda_t = \frac{1}{1+\rho} E_t [\lambda_{t+1} (1 + r_{t+1})]. \quad (21)$$

where $E_t(\cdot)$ is expectation operator conditional on information set, Ω_t .

Now consumer chooses savings so λ_t equals discounted value of what expect λ to be in next period. (Optimal to fully adjust in each period.)

Can show that from (21) can get:

$$\ln \lambda_t = b_t^* + \ln \lambda_{t-1} + \nu_t^* \quad (21')$$

where $b_t^* = \ln\left(\frac{1+\rho}{1+r_t}\right) - \ln\left\{E_t(\exp \nu_t^*)\right\}$ and ν_t^* is forecast error for λ_t . Repeated substitution yields:

$$\ln \lambda_t = \sum_{j=1}^t b_j^* + \ln \lambda_0 + \sum_{j=1}^t \nu_j^* \quad (22)$$

where last component indicates the revisions in λ that occurs over life cycle as a result of shocks to w , p and r .

3.2 Estimation Issues

Altonji (1986) assumes that ε_{1it} (preference shocks in (18)) are orthogonal to Ω_t .

Then without non-participation “problems,” two estimation strategies:

Strategy I:

$$\ln H_t = f_i + b^* t + \delta \ln w_t + z_t \beta_1 + \eta_{it} \quad (23)$$

where $f_i = \delta \ln \lambda_0$, b^* approximates the drift in forecast errors as well as other stuff and

$$\eta_{it} = \varepsilon_{it} + \sum_{j=1}^t v_j^*$$

Now take first-differences to get:

$$\Delta \ln H_t = a_0 + b^* + \delta \Delta \ln w_t + \Delta z_t \beta_1 + \Delta \eta_{it} + v_{it} \quad (24)$$

but now we have to instrument for $\Delta \ln w_t$ since it is correlated with v_{it} .

Strategy II:

Use data on consumption, c_{it} to account for λ_t since:

$$\lambda_{it} = U_c(c_{it}, L_{it}) / p_t \quad (25)$$

Assuming functional form for $U_c(\cdot, \cdot)$ (contemporaneously separable) like

$$U_c(c_{it}, L_{it}) = d_{it} c_{it}^{\gamma-1}$$

we get

$$\ln H_t = \delta(\gamma - 1) \ln c_{it} + \delta \ln \left[\frac{1 + r_t}{1 + \rho} \right] + \delta \ln w_t + z_t \beta_1 + \eta_{it} \quad (26)$$

and now can estimate (26) using instrumental variables for $\ln c_{it}$ and $\ln w_t$.

Above strategy allows us to estimate δ , intertemporal substitution effect in Frisch Demand sense.

What about estimating other wage effects?

This is, in principle, more complicated, given that λ_{it} is evolving stochastically and getting a sense of how changes in wages are affecting it becomes murkier.